AD 451506

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Technical Note N-591

ARCHING IN SOIL DUE TO THE DEFLECTION OF A RIGID HORIZONTAL STRIP

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October 1964

451506

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CONTENTS

NOTATIONS

A,B,C,D	Constants
A(i)	Nondimensionalized pressure $= \frac{p_y^{(i)}}{pb}$
A(i)(J)	Notation for $\left[\frac{b}{ih}\left(1+\frac{x}{b}\right)\right]^J$
A(i)2(J)	$\left\{1+\left[\frac{b}{ih}\left(1+\frac{x}{b}\right)\right]^2\right\}^J$
B(i)(J)	Notation for $\left[\frac{b}{ih}\left(1-\frac{x}{b}\right)\right]^J$
B(i)2(J)	$\left\{1+\left[\frac{b}{ih}\left(1-\frac{x}{b}\right)\right]^2\right\}^J$
b	Half-width of rigid base
C_1, C_2, C_3, C_4	Constants
C(i)(J)	Notation for $\left[\frac{b}{ih}\left(1+\frac{x_{cr}}{b}\right)\right]^J$
C(i)2(J)	$\left\{1+\left[\frac{b}{ih}\left(1+\frac{x_{cr}}{b}\right)\right]^2\right\}^J$
D(i)(J)	Notation for $\left[\frac{b}{ih}\left(1-\frac{x_{cr}}{b}\right)\right]^J$
D(i)2(J)	$\left\{1+\left[\frac{b}{ih}\left(1-\frac{x_{cr}}{b}\right)\right]^2\right\}^J$
d	Displacement of the rigid strip
E	Modulus of elasticity of the soil
h	Depth of soil
i, J	Indices of a variable
$p_y^{(i)}$	Pressure on base $= \int_0^{x_{cr}} \sigma_{y,o}^{(i)}\, dx$

p	Total pressure on the horizontal strip with no deflection
p_o	Overpressure on the soil
R	Amount of arching
S_i	Nondimensional stress = $\dfrac{\sigma^{(i)}_{y,o}}{E\,\frac{d}{h}}$
v	The displacement in the y direction
x_{cr}	Critical distance from the center of the strip at which the net pressure on the strip becomes tensile
$\sigma_x, \sigma_y, \tau_{xy}$	Stresses on an element in the x - y plane
$\epsilon_x, \epsilon_y, \gamma_{xy}$	Strain components in x -y plane
β	Elastic constant $\dfrac{1 - \mu^2}{E}$
ρ	Elastic constant $\dfrac{\mu(1 + \mu)}{E}$
μ	Poisson's ratio
α, λ	Variables of integration
γ	Density of soil
φ	Airy's stress function

INTRODUCTION

If the foundation of an underground structure settles, the pressure transmitted to the structure is reduced with a corresponding increase of pressure in the neighboring soil. This phenomenon is known as arching. Arching due to deflection of a rigid horizontal strip or base has been discussed by Terzaghi,[1] and formulas for the pressure transmitted to the strip are given based on assumed failure planes. The magnitude of the displacement to produce these failure planes is not known. If the deflection of the strip is less than a critical value, failure planes will not develop, and the arching formulas cannot be used effectively. At zero displacement, the pressure on the strip is equal to the pressure of the soil above the strip plus any additional overpressure acting on the soil. The pressure on the strip decreases as the displacement is increased. It should be possible to displace the strip to a critical value such that all the pressure acting on it is transferred to the neighboring soil. The objectives of this study are to establish the limits of the critical displacement and to find the amount of arching and the configuration of the pressure distribution on the base as a function of deflection.

W. D. Finn[2] has treated various problems dealing with stresses in idealized soil media subjected to different types of boundary conditions. One such problem deals with the stresses in soils due to the deflection of a rigid horizontal strip. The depth of the soil was taken as infinity, which imposes a restriction in adapting the solution to practical problems.

The present study deals with the stresses in a soil field of finite depth, h, due to deflection of a rigid strip of width $2b$. The soil mass is assumed to be a homogeneous, elastic, isotropic medium subjected to high overpressures. If there is no deflection of the horizontal strip, the pressure, p, transmitted to the strip will be equal to the overpressure, p_o, plus the pressure of the soil above the base, γh, where γ is the density of the soil. However, if the strip is displaced by an amount d, the pressure transmitted to it will be reduced. Assuming that the principle of superposition is valid, the amount of arching (the amount of pressure that is transferred to the neighboring soil or the reduction of pressure on the strip) is equal to the amount of tensile forces on the base due to the displacement d. However, as the strip deflects, zones of very high tensile stresses form toward the edges because of the discontinuous displacement. Thus, when the overpressure is superimposed on the tensile stress field, there will still be residual tensile stresses toward the edges of the base. Since the soil media cannot be expected to transmit tensile stresses, these stresses are not considered to contribute to arching. This condition is specified when computing the amount of arching.

ANALYTICAL DEVELOPMENT

Procedure

Figure 1 represents a section through a soil mass of depth h and infinite width subjected to an overpressure p_o. The distance in the z direction perpendicular to the x-y plane is considered as infinity. The width of the rigid horizontal strip is taken as 2b, and the amount of plate deflection given by d. The total pressure on the strip with conditions given in Case (a), Figure 1, can be considered to be equivalent to the superposition of Cases (b) and (c). Case (b) represents a uniform compression on the base exerted by the overpressure and the soil above the base ($p = p_o + \gamma h$), with no deflection of the rigid strip. The distribution of pressure in Case (b) is indicated by line 12. The tensile stress distribution due to the displacement d alone is assumed to be given by 34567 of Case (c). The tensile stresses at the edge are infinite due to the discontinuity of displacement. The pressure distribution for Case (a) can be assumed to be given by the superposition of Cases (b) and (c) for small displacements of the strip.

It is to be noted that the tensile stresses due to the base displacement reach the value of the maximum compression, p, at some critical distance, $\pm x_{cr}$, from the center of the base. Beyond this region the net pressure on the base is tensile. These resultant tensile stresses in the region beyond $\pm x_{cr}$ from the center of the base are not considered effective; thus, the net compressive force acting on the strip is given by the area 456.

The express objectives of this study are to find the distribution of tensile stresses as shown in Case (c) by 34567, to determine the distance $\pm x_{cr}$ at which the resultant pressures become tensile, and to find the amount of arching as shown in Case (a) by 14562.

Evaluation of Stress Function

Since it is assumed that there are no strains in the z direction, the problem can be considered as one of "plane strain," and the appropriate equations of the theory of elasticity can be used.[3] The positive directions of the stresses σ_{xy}, σ_y, and τ_{xy} are shown in Figure 1. Since the surface is free of applied pressure, the boundary conditions for Case (c) at y = h are given by

$$\sigma_y = 0 \tag{1}$$

$$\tau_{xy} = 0 \tag{2}$$

The frictional resistance at the base, y = 0, can be assumed to prevent any elongation in the x direction at that level. Thus, the strain in the x direction at y = 0 is

$$\epsilon_{x,y=0} = \beta\sigma_x - \rho\sigma_y = 0 \tag{3}$$

where

$$\beta = \frac{1-\mu^2}{E} \quad \text{and} \quad \rho = \frac{\mu\ (1+\mu)}{E} \tag{4}$$

E is the modulus of elasticity of the soil media, and μ is Poisson's ratio.

The displacement, v, in the y direction at the boundary y = 0 is given by

$$\begin{aligned} v &= -d \text{ for } -b \le x \le +b \\ &= 0 \text{ for } x<-b \quad \text{and} \quad x>+b \end{aligned} \tag{5}$$

The displacement at the boundary can be expressed in integral form by using the Fourier cosine integral:

$$v(x) = \frac{2}{\pi} \int_0^\infty \cos \alpha x \, d\alpha \int_c^\infty v(\lambda) \cos \alpha\lambda \, d\lambda \tag{6}$$

Thus, the displacement at y = 0 as defined by Equation 5 can be obtained as

$$v(x) = -\frac{2d}{\pi} \int_0^\infty \frac{\sin \alpha b}{\alpha} \cos \alpha x \, d\alpha \tag{7}$$

where α and λ are variables of integration.

The displacement, v, can be expressed in terms of strain ϵ_y in the y direction as

$$\epsilon_y = \frac{\partial v}{\partial y} = \beta\sigma_y - \rho\sigma_x \tag{8}$$

$$\text{and } v = \int \epsilon_y dy + g(x) \tag{9}$$

To solve for the various stresses in the media, with the given boundary conditions, a stress function[2] in the following form is assumed:

$$\varphi = \int_0^\infty \frac{1}{\alpha^2} \left[Ae^{\alpha y} + B\alpha e^{\alpha y} + Ce^{-\alpha y} + D\alpha e^{-\alpha y} \right] \cos \alpha x \quad dx \tag{10}$$

where A, B, C, and D are constants.

It can be shown that the above stress function, φ , satisfies the biharmonic equation

$$\nabla^4 \varphi = \frac{\partial^4 \varphi}{\partial x^4} + 2 \frac{\partial^4 \varphi}{\partial x^2 \partial y^2} + \frac{\partial^4 \varphi}{\partial y^4} = 0 \tag{11}$$

The stresses in the media are related to the stress function by the following relations:

$$\sigma_x = \frac{\partial^2 \varphi}{\partial x^2}$$

$$\sigma_y = \frac{\partial^2 \varphi}{\partial y^2} \tag{12}$$

$$\tau_{xy} = - \frac{\partial^2 \varphi}{\partial x \, \partial y}$$

The constants A, B, C, and D are found by using the boundary conditions (1), (2), (3), and (7) as specified above. The functions g(x) in Equation 9 can be shown to be zero after appropriate substitutions and integrations using the condition that $v = 0$ for $h \rightarrow \infty$. The relations for the constants A, B, C, and D are given below:

$$A = \left[C \, (-2\alpha h - 1) + D \, (-2\alpha^2 h^2) \right] e^{-2\alpha h} \tag{13}$$

$$B = \left[2C + D \, (2\alpha h - 1) \right] e^{-2\alpha h} \tag{14}$$

$$C = D\left[\frac{e^{-2\alpha h}\left\{(-2\alpha^2h^2 + 4\alpha h - 2)\,\beta - 2\alpha^2h^2\rho\right\} - 2\,\beta}{e^{-2\alpha h}\left\{(2\alpha h - 3)\beta + (2\alpha h + 1)\rho\right\} - \beta - \rho}\right] \tag{15}$$

$$D = \frac{-2d}{\pi}\left[\frac{\left[e^{-2\alpha h}\left\{(2\alpha h - 3)\beta + (2\alpha h + 1)\rho\right\} - \beta - \rho\right]}{\begin{array}{l}\left[e^{-4\alpha h}\left\{-3\beta^2 - 2\beta\rho + \rho^2\right\}\right. \\ +e^{-2\alpha h}\left\{(-4\alpha^2h^2 - 10)\,\beta^2 + (-8\alpha^2h^2 + 4)\rho\beta\right. \\ \qquad\left. +(-4\alpha^2h^2 - 2)\,\rho^2\right\} \\ \left. -3\beta^2 - 2\beta\rho + \rho^2\right]\end{array}}\right]\sin\alpha b \tag{16}$$

Substitution of the expressions for A, B, C, and D in Equation 10 and subsequent simplification will result in an expression for the stress function. The stresses anywhere in the medium can be found by using Equation 12. However, since the σ_y stresses over the base are of main interest, only these are evaluated in the next section.

Pressure Distribution Across the Width of the Strip

The expression for the stress σ_y is obtained by substituting the stress function in Equation 12 as follows:

$$\sigma_y = \int_0^\infty - \frac{2d}{\pi} \left[\frac{\begin{array}{l} e^{\alpha(y-4h)} \Big[\left\{ \alpha\,(\beta\,y + \rho\,y) - 2\beta \right\} \\ + e^{\alpha(y-2h)} \Big[\alpha^2 \left\{ 2\beta\,(-h^2 + hy) + 2\rho\,(-h^2 + hy) \right\} \\ \qquad + \alpha \left\{ \beta\,(-4h + 3y) - y\,\rho \right\} - 2\beta \Big] \\ + e^{\alpha(-y-2h)} \Big[\alpha^2 \left\{ 2\beta\,(h^2 - hy) + 2\rho\,(h^2 - hy) \right\} \\ \qquad + \alpha \left\{ \beta\,(-4h + 3y) - y\,\rho \right\} + 2\beta \Big] \\ + e^{-\alpha y} \Big[\alpha \left\{ \beta\,y + \rho y \right\} + 2\beta \Big] \end{array}}{\begin{array}{l} e^{-4\alpha h} \left\{ -3\beta^2 - 2\rho B + \rho^2 \right\} \\ + e^{-2\alpha h} \Big[\alpha^2 \left\{ -4\beta^2 h^2 - 8\rho\beta h^2 - 4\rho^2 h^2 \right\} \\ \qquad + \left\{ -10\beta^2 + 4\beta\rho - 2\rho^2 \right\} \Big] \\ + (-3\beta^2 - 2\beta\rho + \rho^2) \end{array}} \right] \begin{array}{l} \sin b\alpha \\ \cos x\alpha \\ d\alpha \end{array} \quad (17)$$

This expression for σ_y can be specialized to obtain the pressure distribution across the horizontal strip by integrating and then substituting y = 0 in the final results. However, with some care, y = 0 can be substituted in some terms of the numerator of the integrand before performing the integration, and the following integral is obtained for $\sigma_{y,o}$ along y = 0:

$$\sigma_{y,o} = -2\beta C_3 \int_o^\infty \left[\frac{e^{-4\alpha h} + 4\alpha h e^{-2\alpha h} - e^{-\alpha y}}{e^{-4\alpha h} + e^{-2\alpha h}\,(4C_1\alpha^2 h^2 + C_2) + 1} \right] 2 \sin b\alpha \cos x\alpha \, d\alpha \tag{18}$$

where

$$C_1 = \left[\frac{-\beta^2 - 2\beta\rho - \rho^2}{-3\beta^2 - 2\beta\rho + \rho^2} \right]$$

$$C_2 = \left[\frac{-10\beta^2 + 4\beta\rho - 2\rho^2}{-3\beta^2 - 2\beta\rho + \rho^2} \right]$$

$$C_3 = -\frac{d}{\pi} \left[\frac{1}{-3\beta^2 - 2\beta\rho + \rho^2} \right]$$

$$C_4 dE = -2\beta C_3$$

$$\text{or } C_4 = \frac{2\beta}{\pi E\,(-3\beta^2 - 2\beta\rho + \rho^2)}$$

C_1, C_2, and C_4 are dimensionless constants, and C_3 has the units of pounds per inch.

The integration of the expression given by Equation 18 is achieved by expanding the denominator in a series. Thereafter, all but the first six terms of this alternating series are neglected. The resulting expression is shown below:

$$\sigma_{y,o} = C_4 dE \int_0^\infty \Bigg\{ 1 - e^{-2\alpha h} \left(4C_1\alpha^2 h^2 + C_2\right)$$

$$+ e^{-4\alpha h} \left[\left(4C_1\alpha^2 h^2 + C_2\right)^2 - 1 \right]$$

$$- e^{-6\alpha h} \left[\left(4C_1\alpha^2 h^2 + C_2\right)^3 - 2 \left(4C_1\alpha^2 h^2 + C_2\right) \right]$$

$$+ e^{-8\alpha h} \left[\left(4C_1\alpha^2 h^2 + C_2\right)^4 - 3 \left(4C_1\alpha^2 h^2 + C_2\right)^2 + 1 \right]$$

$$- e^{-10\alpha h} \left[\left(4C_1\alpha^2 h^2 + C_2\right)^5 - 4 \left(4C_1\alpha^2 h^2 + C_2\right)^3 + 3 \left(4C_1\alpha^2 h^2 + C_2\right) \right]$$

$$+ e^{-12\alpha h} \left[\qquad \qquad \right] + \ . \ . \ . \ . \Bigg\} \left\{ e^{-4\alpha h} + 4\alpha h \, e^{-2\alpha h} - e^{-\alpha y} \right\}$$

$$\left\{ 2 \sin b\alpha \cos x\alpha \right\} d\alpha \qquad (19)$$

or

$$\sigma_{y,o} = \Sigma \, \sigma_{y,o}^{(i)}$$

$$= \sigma_{y,o}^{(1)} + \sigma_{y,o}^{(2)} + \sigma_{y,o}^{(3)} + \sigma_{y,o}^{(4)} + \sigma_{y,o}^{(5)} + \sigma_{y,o}^{(6)} + \ . \ . \ . \qquad (20)$$

where

$$\sigma_{y,o}^{(1)} = C_4 \, dE \int_0^\infty \Big[1 \Big] \left[e^{-4\alpha h} + 4\alpha h \, e^{-2\alpha h} - e^{-\alpha y} \right] \Big[2 \sin b\alpha \cos x\alpha \Big] d\alpha$$

$$\sigma^{(2)}_{y,o} = -C_4\, dE \int_o^\infty e^{-2\alpha h}\left[4C_1\alpha^2h^2 + C_2\right]\left[e^{-4\alpha h} + 4\alpha h\, e^{-2\alpha h} - e^{-\alpha y}\right]$$

$$\left[2 \sin b\alpha \cos x\alpha\right] d\alpha$$

$$\sigma^{(3)}_{y,o} = C_4\, dE \int_o^\infty e^{-4\alpha h}\left[(4C_1\alpha^2h^2 + C_2)^2 - 1\right]$$

$$\left[e^{-4\alpha h} + 4\alpha h\, e^{-2\alpha h} - e^{-\alpha y}\right]$$

$$\left[2 \sin b\alpha \cos x\alpha\right] d\alpha$$

$$\sigma^{(4)}_{y,o} = -C_4\, dE \int_o^\infty e^{-6\alpha h}\left[(4C_1\alpha^2h^2 + C_2)^3 - 2\,(4C_1\alpha^2h^2 + C_2)\right]$$

$$\left[e^{-4\alpha h} + 4\alpha h\, e^{-2\alpha h} - e^{-\alpha y}\right]$$

$$\left[2 \sin b\alpha \cos x\alpha\right] d\alpha$$

$$\sigma^{(5)}_{y,o} = C_4\, dE \int_o^\infty e^{-8\alpha h}\left[(4C_1\alpha^2h^2 + C_2)^4 - 3\,(4C_1\alpha^2h^2 + C_2)^2 + 1\right]$$

$$\left[e^{-4\alpha h} + 4\alpha h\, e^{-2\alpha h} - e^{-\alpha y}\right]$$

$$\left[2 \sin b\alpha \cos x\alpha\right] d\alpha$$

$$\sigma_{y,o}^{(6)} = - C_4 \, dE \int_o^\infty e^{-10\alpha h} \Big[(4C_1\alpha^2h^2 + C_2)^5 - 4 (4C_1\alpha^2h^2 + C_2)^3$$

$$+ 3(4C_1\alpha^2h^2 + C_2) \Big]$$

$$\Big[e^{-4\alpha h} + 4\alpha h \, e^{-2\alpha h} - e^{-\alpha y} \Big]$$

$$\Big[2 \sin b\alpha \cos x\alpha \Big] d\alpha \tag{21}$$

The expressions for $\sigma_{y,o}^{(i)}$ are evaluated individually and then y is set to zero. In the expression for $\sigma_{y,o}^{(1)}$, if the value of zero for y is substituted in the integrand, the improper integral

$$\int_o^\infty \sin b\alpha \cos x\alpha \, d\alpha$$

does not exist.[4] However, with the term $e^{-\alpha y}$ in the integrand, the value of the integral is given by

$$- C_4 \, dE \int_o^\infty e^{-\alpha y} (2 \sin b\alpha \cos x\alpha) d\alpha = - C_4 \, dE \Bigg[\frac{b + x}{y^2 + (b + x)^2}$$

$$+ \frac{b - x}{y^2 + (b - x)^2} \Bigg] \tag{22a}$$

and the limiting value as $y \to 0$ is given by

$$- C_4 \, dE \left[\frac{1}{b + x} + \frac{1}{b - x} \right] \tag{22b}$$

The final expressions for $\sigma_{y,o}^{(i)}$ are obtained after the evaluation of the integrals in Equation 21. These expressions for $\sigma_{y,o}^{(i)}$ are non-dimensionalized by E(d/h) to give S_i's. Thus,

$$S_i = \frac{\sigma_{y,o}^{(i)}}{E\,\frac{d}{h}} \tag{23}$$

It can be noted that d/h indicates the uniform strain in the media if the yielding strip extends to infinity, and E(d/h) represents the corresponding uniform stress in the media. The expressions for S_i's are given in Appendix A-I.

From the preceding it can be seen that

$$S_i = S_i\ (b/h,\ x/b,\ \mu) \tag{24}$$

and

$$S = \sum_{i=1}^{6} S_i$$

A digital computer program was written to evaluate the six terms of the series for several values of the parameters as shown below:

b/h =	0.05	0.4	0.8
	0.1	0.5	0.9
	0.2	0.6	1.0
	0.3	0.7	

μ = 0.1, 0.25, 0.3333, 0.5

x/b = 0 to 0.95 at intervals of 0.05

Selected tabulated results and a consideration of the errors of the solutions are given in Appendix A-II. For practical purposes, the accuracy is deemed quite adequate.

The results showing S versus x/b for different values of b/h and μ are given in Figures 2 through 5. In all these cases the value of S reaches infinity at x/b = 1.0 due to the discontinuity at the boundary. For smaller values of b/h, the curve in the central portion if flatter than for larger values of b/h. For the extreme case of b/h = 1.0 and μ = 0.5, the pressure at the center of the base (at x/b = 0) is very small in compression, whereas for other portions of the strip the stresses are tensile. Comparing the plots for various values of μ, it can be seen that there is little difference in the stress distribution except for the extreme cases of b/h = 1.0.

Computation of Arching

The intensity of the resultant pressure at any point on the strip when the soil of depth h is subjected to an overpressure of p_o is given by

$$p_{resultant} = p_o + \gamma h - \sigma_{y,o} = p - \sigma_{y,o} \qquad (25)$$

Nondimensionalizing by E(d/h), Equation 25 can be rewritten as

$$\frac{\left(p_{resultant}\right) h}{dE} = \frac{ph}{dE} - S \qquad (26)$$

The term $p_{resultant}$ in Equations 25 and 26 should be considered as equal to zero for negative values of (ph/dE) - S.

The ratio $\pm x_{cr}/b$, for which the resultant pressure becomes negative, can be found for any given set of b/h, μ, and ph/dE parameters. Thus, the total tensile forces, R, over the half-width of strip due to displacement, d, can be obtained by integrating six stress terms as given by Equation 30 (Appendix A) with respect to x, evaluating the definite integral from 0 to $+ x_{cr}$, and adding the tensile forces from x_{cr} to b.. Thus, the amount of arching, that is the total pressure transferred to the neighboring soil, is given by

$$R = \Sigma\, p_y^{(i)} + p\,(b - x_{cr}) \qquad (27)$$

where

$$p_y^{(i)} = \int_o^{x_{cr}} \sigma_{y,o}^{(i)}\, dx$$

Thus,

$$\text{percentage of arching} = \frac{R}{pb}\, 100 \qquad (28)$$

$$= \left[A + 1 - \frac{x_{cr}}{b} \right] 100$$

where

$$A = \sum_{i=1}^{6} A_i = \sum_{i=1}^{6} \frac{p_y^{(i)}}{pb}$$

$$A_i = A_i \left(\frac{b}{h}, \frac{x_{cr}}{b}, \mu, \frac{pb}{dE}\right)$$

The expressions for A_i's are given in Appendix B-I.

Given the parameters ph/dE, μ, and b/h, a digital computer program was developed to find the value of x_{cr}/b. The program initially assumes a value of x/b = 0, computes the value of S, and compares it with ph/dE. If the value of S is greater than ph/dE, it indicates that the net pressure on the base is tensile and an arching of 100 percent is indicated. If the value of S at x/b = 0 is smaller than ph/dE, then a certain increment is given to x/b and a new value of S (x/b, b/h, μ) is computed and is compared with ph/dE. If the difference between these two is less than or equal to 10^{-5}, that value of x/b is taken as x_{cr}/b. If the difference is greater than 10^{-5}, the program assumes another value of x/b and the process is repeated until the value of x_{cr}/b is reached. For values of x/b approaching 1, the computer takes a very long time to find the value of x_{cr}/b. In cases where the number of iterations exceed more than 100, the computer prints out "number of iterations more than 100 to find x_{cr}" and proceeds with the next problem. After the value of x_{cr}/b is computed, the expressions for the A_i's are evaluated and the percentage of arching is computed using Equation 28. For any problem, the values of b/h, μ, ph/dE, x_{cr}/b, A_1, A_2, A_3, A_4, A_5, A_6, A, and arching are printed. The following values are used for the parameter ph/dE, and the range is considered adequate.

ph/dE =	0.01	0.1	1	10	100	1000
	0.0125	0.125	1.25	12.5	125	
	0.015	0.15	1.5	15	150	
	0.02	0.2	2	20	200	
	0.03	0.3	3	30	300	
	0.04	0.4	4	40	400	
	0.05	0.5	5	50	500	
	0.06	0.6	6	60	600	
	0.07	0.7	7	70	700	
	0.09	0.9	9	90	900	

Selected tabulated results and a study of the errors involved in taking the first six terms of the series are given in Appendix B-II. The errors for the most part are found to be small.

The value of x_{cr}/b is zero when the arching is 100 percent, and at zero percent arching the value of x_{cr}/b is very nearly 1. Figures 6 through 9 indicate the variation of x_{cr}/b with ph/dE for Poisson's ratio equal to 0.1, 0.25, 0.3333, and 0.5.

Figures 10 through 13 indicate the percentage of arching versus the yield parameter ph/dE for different values of b/h and Poisson's ratio, μ. For each value of b/h, the percentage of arching decreases with increasing values of ph/dE.

Comparing the plots of arching versus ph/dE and x_{cr}/b versus ph/dE for different values of Poisson's ratio, it can be seen that the effect of Poisson's ratio can be neglected over a wide range of ph/dE for small values of b/h.

Illustrative Example

It is desired to find the amount of arching developed when a rigid horizontal strip 24 feet wide buried under 17 feet of soil cover undergoes a displacement of 2 inches. The soil is subjected to an overpressure of 100 psi. The modulus of elasticity of the soil, E, is 10,000 psi, Poisson's ratio, μ, is 0.25, and the density of the soil, γ , is 110 pcf. Thus,

$$b = 12 \text{ ft}$$

$$h = 17 \text{ ft}$$

$$b/h = 0.706$$

$$p_o = 100 \text{ psi}$$

$$\gamma = 110 \text{ pcf}$$

$$\gamma h = \frac{110(17)}{144} = 13 \text{ psi}$$

$$p = p_o + \gamma h = 113 \text{ psi}$$

$$\text{yield parameter, } \frac{ph}{dE} = \frac{113(17)(12)}{2(10,000)} \approx 1.15$$

From the plots in Figure 7 the value of $x_{cr}/b = 0.77$ and from Figure 11 the amount of arching can be found to be 52 percent for b/h = 0.706 and ph/dE = 1.15. The distribution of pressure on the base can be obtained from Figure 3. At x/b = 0, the pressure, p, at zero deflection is reduced by $\sigma_{y,o} = 20.6$ psi, where $\sigma_{y,o}$ is obtained from $S = \sigma h/dE = 0.21$. Thus, the net pressure on the base at x/b = 0 is 113 - 20.6 = 92.4 psi, and the pressure reduces to zero at a distance $\pm$ 0.77 x 12 = $\pm$ 9.24 feet from the center of the base.

CONCLUSIONS

The analysis indicates that arching for the case considered here varies from 100 percent to zero percent, depending upon the three parameters, b/h, ph/dE, and μ. However, for practical purposes the effect of Poisson's ratio, μ, can be neglected over a wide range of parameters. The errors found in taking the first six terms of the infinite series are small for a majority of cases. Experimental data to determine the validity of the theory are not available at this time. However, these results were compared with the data obtained for a case very similar to the one considered here, and the trends for the amount of arching are similar to those given here.

ACKNOWLEDGMENTS

The author wishes to express his gratitude to Mr. Jay R. Allgood, Structural Research Engineer, U. S. Naval Civil Engineering Laboratory, for suggesting the problem and for many helpful comments during the course of this investigation. The programs were checked and the results were compiled using the Honeywell 800 Data Processing System at the University of Southern California, Los Angeles. Part of the results were obtained using the IBM 7090 Data Processing system at the Pacific Missile Range, Point Mugu.

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Appendix A

S_i SERIES, EQUATION 24

I. EXPRESSIONS FOR S_i's

The following notations are used when expressing S_i's:

$$A(i)(J) = \left[\frac{b}{ih} \left(1 + \frac{x}{b}\right) \right]^J$$

$$B(i)(J) = \left[\frac{b}{ih} \left(1 - \frac{x}{b}\right) \right]^J$$

$$A(i)2(J) = \left\{ 1 + \left[\frac{b}{ih} \left(1 + \frac{x}{b}\right) \right]^2 \right\}^J \tag{29}$$

$$B(i)2(J) = \left\{ 1 + \left[\frac{b}{ih} \left(1 - \frac{x}{b}\right) \right]^2 \right\}^J$$

The last two notations are used only in the denominators of the expressions for S_i's and $i \geq j-1$. The resulting relations are:

$$S_1 = \frac{\sigma_{y,o}^{(1)}}{E \frac{d}{h}} = C_4 \left[0.25 \left\{ \frac{A41}{A421} + \frac{B41}{B421} \right\} + 1 \left\{ \frac{A11}{A222} + \frac{B11}{B222} \right\} - 1 \left\{ \frac{1}{A11} + \frac{1}{B11} \right\} \right] \tag{30a}$$

$$S_2 = \frac{\sigma_{y,o}^{(2)}}{E\,\frac{d}{h}} = C_4 \Bigg[C_1 \Bigg\{ -\frac{1}{27} \left[\frac{3(A61) - A63}{A623} + \frac{3(B61) - B63}{B623} \right]$$

$$- 1.5 \left[\frac{A41 - A43}{A424} + \frac{B41 - B43}{A424} \right]$$

$$+ 1 \left[\frac{3(A21) - A23}{A223} + \frac{3(B21) - B23}{B223} \right] \Bigg\}$$

$$+ C_2 \Bigg\{ - \frac{1}{6} \left[\frac{A61}{A621} + \frac{B61}{B621} \right]$$

$$- 0.5 \left[\frac{A41}{A422} + \frac{B41}{B422} \right]$$

$$+ 0.5 \left[\frac{A21}{A221} + \frac{B21}{B221} \right] \Bigg\} \Bigg] \tag{30b}$$

$$S_3 = \frac{\sigma_{y,o}^{(3)}}{E\,\frac{d}{h}}$$

$$= C_4 \Bigg[C_1^2 \Bigg\{ 0.0117188 \Bigg[\frac{5(A81) - 10(A83) + A85}{A825}$$

$$+ \frac{5(B81) - 10(B83) + B85}{B825} \Bigg]$$

$$+ 0.1646091 \Bigg[\frac{6(A61) - 20(A63) + 6(A65)}{A626}$$

$$
+ \frac{6(B61) - 20(B63) + 6(B65)}{B626} \Bigg]
$$

$$
- 0.375 \Bigg[\frac{5(A41) - 10(A43) + A45}{A425}
$$

$$
+ \frac{5(B41) - 10(B43) + B45}{B425} \Bigg] \Bigg\}
$$

$$
+ C_1C_2 \Bigg\{ 0.03125 \Bigg[\frac{3(A81) - A83}{A823} + \frac{3(B81) - B83}{B823} \Bigg]
$$

$$
+ 0.1481481 \Bigg[\frac{4(A61) - 4(A63)}{A624} + \frac{4(B61) - 4(B63)}{B624} \Bigg]
$$

$$
- 0.25 \Bigg[\frac{3(A41) - A43}{A423} + \frac{3(B41) - B43}{B423} \Bigg] \Bigg\}
$$

$$
+ (C_2^{\,2} - 1) \Bigg\{ 0.125 \Bigg[\frac{A81}{A821} + \frac{B81}{B821} \Bigg]
$$

$$
+ 0.1111111 \Bigg[\frac{2(A61)}{A622} + \frac{2(B61)}{B622} \Bigg]
$$

$$
- 0.25 \Bigg[\frac{A41}{A421} + \frac{B41}{B421} \Bigg] \Bigg\} \Bigg] \qquad (30c)
$$

$$S_4 = \frac{\sigma_{y,o}^{(4)}}{E\,\frac{d}{h}}$$

$$= C_4 \Bigg[C_1^{\,3} \Bigg\{ - 0.0046080 \left[\frac{7(A101) - 35(A103) + 21(A105) - A107}{A1027} \right.$$

$$\left. + \frac{7(B101) - 35(B103) + 21(B105) - B107}{B1027} \right]$$

$$- 0.0769043 \left[\frac{8(A81) - 56(A83) + 56(A85) - 8(A87)}{A828} \right.$$

$$\left. + \frac{8(B81) - 56(B83) + 56(B85) - 8(B87)}{B828} \right]$$

$$+ 0.1646091 \left[\frac{7(A61) - 35(A63) + 21(A65) - A67}{A627} \right.$$

$$\left. + \frac{7(B61) - 35(B63) + 21(B65) - B67}{B627} \right] \Bigg\}$$

$$+ C_1^{\,2} C_2 \Bigg\{ - 0.01152 \left[\frac{5(A101) - 10(A103) + A105}{A1025} \right.$$

$$\left. + \frac{5(B101) - 10(B103) + B105}{B1025} \right]$$

$$- 0.0878906 \left[\frac{6(A81) - 20(A83) + 6(A85)}{A826} \right.$$

$$+ \frac{6(B81) - 20(B83) + 6(B85)}{B826} \Bigg]$$

$$+ 0.1481482 \Bigg[\frac{5(A61) - 10(A63) + A65}{A625}$$

$$+ \frac{5(B61) - 10(B63) + B65}{B625} \Bigg] \Bigg\}$$

$$+ (3C_1C_2^{\ 2} - 2C_1) \Bigg\{ - 0.008 \Bigg[\frac{3(A101) - A103}{A1023} + \frac{3(B101) - B103}{B1023} \Bigg]$$

$$- 0.0234375 \Bigg[\frac{4(A81) - 4(A83)}{A824} + \frac{4(B81) - 4(B83)}{B824} \Bigg]$$

$$+ 0.0370370 \Bigg[\frac{3(A61) - A63}{A623} + \frac{3(B61) - B63}{B623} \Bigg] \Bigg\}$$

$$+ (C_2^{\ 3} - 2C_2) \Bigg\{ - 0.10 \Bigg[\frac{A101}{A1021} + \frac{B101}{B1021} \Bigg]$$

$$- 0.0625 \Bigg[\frac{2(A81)}{A822} + \frac{2(B81)}{B822} \Bigg]$$

$$+ 0.1666667 \Bigg[\frac{A61}{A621} + \frac{B61}{B621} \Bigg] \Bigg\} \Bigg] \tag{30d}$$

$$S_5 = \frac{\sigma_{y,o}^{(5)}}{E\frac{d}{h}} = C_4 \Bigg[C_1^{\,4} \Bigg\{ 0.0020005 \left[\frac{9(A121) - 84(A123) + 126(A125) - 36(A127) + A129}{A1229} \right.$$

$$\left. + \frac{9(B121) - 84(B123) + 126(B125) - 36(B127) + B129}{B1229} \right]$$

$$+ 0.0371589 \left[\frac{10(A101) - 120(A103) + 252(A105) - 120(A107) + 10(A109)}{A10210} \right.$$

$$\left. + \frac{10(B101) - 120(B103) + 252(B105) - 120(B107) + 10(B109)}{B10210} \right]$$

$$- 0.0769043 \left[\frac{9(A81) - 84(A83) + 126(A85) - 36(A87) + A89}{A829} \right.$$

$$\left. + \frac{9(B81) - 84(B83) + 126(B85) - 36(B87) + B89}{B829} \right] \Bigg\}$$

$$+ C_1^{\,3} C_2 \Bigg\{ 0.0051440 \left[\frac{7(A121) - 35(A123) + 21(A125) - A127}{A1227} \right.$$

$$\left. + \frac{7(B121) - 35(B123) + 21(B125) - B127}{B1227} \right]$$

$$+ 0.0516096 \left[\frac{8(A101) - 56(A103) + 56(A105) - 8(A107)}{A1028} \right.$$

$$\left. + \frac{8(B101) - 56(B103) + 56(B105) - 8(B107)}{B1028} \right]$$

$$- 0.0878906 \left[\frac{7(A81) - 35(A83) + 21(A85) - A87}{A827} \right.$$

$$\left. \left. + \frac{7(B81) - 35(B83) + 21(B85) - B87}{B827} \right] \right\}$$

$$+ (2C_2^{\,2} - 1)\ C_1^{\,2} \left\{ 0.0046296 \left[\frac{5(A121) - 10(A123) + A125}{A1225} + \frac{5(B121) - 10(B123) + B125}{B1225} \right] \right.$$

$$+ 0.02304 \left[\frac{6(A101) - 20(A103) + 6(A105)}{A1026} + \frac{6(B101) - 20(B103) + 6(B105)}{B1026} \right]$$

$$\left. - 0.0351563 \left[\frac{5(A81) - 10(A83) + A85}{A825} + \frac{5(B81) - 10(B83) + B85}{B825} \right] \right\}$$

$$+ (2C_2^2 - 3)\, C_1 C_2 \Big\{ 0.0092593 \left[\frac{3(A121) - A123}{A1223} + \frac{3(B121) - B123}{B1223} \right]$$

$$+ 0.0192 \left[\frac{4(A101) - 4(A103)}{A1024} + \frac{4(B101) - 4(B103)}{B1024} \right]$$

$$- 0.03125 \left[\frac{3(A81) - A83}{A823} + \frac{3(B81) - B83}{B823} \right] \Big\}$$

$$+ (C_2^4 - 3C_2 + 1) \Big\{ 0.0833333 \left[\frac{A121}{A1221} + \frac{B121}{B1221} \right]$$

$$+ 0.04 \left[\frac{2(A101)}{A1022} + \frac{2(B101)}{B1022} \right]$$

$$- 0.125 \left[\frac{A81}{A821} + \frac{B81}{B821} \right] \Big\} \Bigg] \qquad (30e)$$

$$S_6 = \frac{\sigma_{y,o}^{(6)}}{E \frac{d}{h}}$$

$$= C_4 \Bigg[C_1^5 \Big\{ - 0.0009176 \left[\frac{11(A141) - 165(A143) + 462(A145) - 330(A147) + 55(A149) - A1411}{A14211} \right.$$

$$+ \frac{11(B141) - 165(B143) + 462(B145) - 330(B147) + 55(B149) - B1411}{B14211} \Bigg]$$

$$- 0.0183375 \Bigg[\frac{12(A121) - 220(A123) + 792(A125) - 792(A127) + 220(A129) - 12(A1211)}{A12212}$$

$$+ \frac{12(B121) - 220(B123) + 792(B125) - 792(B127) + 220(B129) - 12(B1211)}{B12212} \Bigg]$$

$$+ 0.0371589 \Bigg[\frac{11(A101) - 165(A103) + 462(A105) - 330(A107) + 55(A109) - A1011}{A10211}$$

$$+ \frac{11(B101) - 165(B103) + 462(B105) - 330(B107) + 55(B109) - B1011}{B10211} \Bigg] \Bigg\}$$

$$+ C_1{}^4C_2 \Bigg\{ - 0.0024979 \Bigg[\frac{9(A141) - 84(A143) + 126(A145) - 36(A147) + A149}{A1429}$$

$$+ \frac{9(B141) - 84(B143) + 126(B145) - 36(B147) + B149}{B1429} \Bigg]$$

$$- 0.0300069 \Bigg[\frac{10(A121) - 120(A123) + 252(A125) - 120(A127) + 10(A129)}{A12210}$$

$$+ \frac{10(B121) - 120(B123) + 252(B125) - 120(B127) + 10(B129)}{B12210} \Bigg]$$

$$+ 0.0516096 \Bigg[\frac{9(A101) - 84(A103) + 126(A105) - 36(A107) + A109}{A1029}$$

$$+ \frac{9(B101) - 84(B103) + 126(B105) - 36(B107) + B109}{B1029} \Bigg] \Bigg\}$$

$$+ C_1^{\,3}(5C_2^{\,2} - 2) \Bigg\{ - 0.0008743 \Bigg[\frac{7(A141) - 35(A143) + 21(A145) - A147}{A1427}$$

$$+ \frac{7(B141) - 35(B143) + 21(B145) - B147}{B1427} \Bigg]$$

$$- 0.0060014 \Bigg[\frac{8(A121) - 56(A123) + 56(A125) - 8(A127)}{A1228}$$

$$+ \frac{8(B121) - 56(B123) + 56(B125) - 8(B127)}{B1228} \Bigg]$$

$$+ 0.009216 \Bigg[\frac{7(A101) - 35(A103) + 21(A105) - A107}{A1027}$$

$$+ \frac{7(B101) - 35(B103) + 21(B105) - B107}{B1027}\Bigg]\Bigg\}$$

$$+ C_1^{\,2}(5C_2^{\,3} - 6C_2)\Bigg\{ - 0.001428\left[\frac{5(A141) - 10(A143) + A145}{A1425} + \frac{5(B141) - 10(B143) + B145}{B1425}\right]$$

$$- 0.005144\left[\frac{6(A121) - 20(A123) + 6(A125)}{A1226} + \frac{6(B121) - 20(B123) + 6(B125)}{B1226}\right]$$

$$+ 0.00768\left[\frac{5(A101) - 10(A103) + A105}{A1025} + \frac{5(B101) - 10(B103) + B105}{B1025}\right]\Bigg\}$$

$$+ C_1(5C_2^{\,4} - 12C_2^{\,2} + 3)\Bigg\{ - 0.0029155\left[\frac{3(A141) - A143}{A1423} + \frac{3(B141) - B143}{B1423}\right]$$

$$- 0.0046296\left[\frac{4(A121) - 4(A123)}{A1224} + \frac{4(B121) - 4(B123)}{B1224}\right]$$

$$+ 0.008\left[\frac{3(A101) - A103}{A1023} + \frac{3(B101) - B103}{B1023}\right]\Bigg\}$$

$$+ (C_2^5 - 4C_2^3 + 3C_2) \Bigg\{ - 0.0714286 \left[\frac{A141}{A1421} + \frac{B141}{B1421} \right]$$

$$- 0.0277778 \left[\frac{2(A121)}{A1222} + \frac{2(B121)}{B1222} \right]$$

$$+ 0.10 \left[\frac{A101}{A1021} + \frac{B101}{B1021} \right] \Bigg\} \Bigg] \qquad (30f)$$

II. ACCURACY OF THE SOLUTIONS

A digital computer evaluation of Equation 24 was made of the first six S_i terms for the parameters b/h, μ, and x/b to study the possible errors in the alternating series. Typical results are given in Tables I through VII for several values of b/h.

It is extremely difficult to prove the convergence of the series. If the sixth term is sufficiently small compared to the first term, the sum of the series is assumed sufficiently accurate for engineering purposes.

For b/h = 1.0, μ = 0.1, and x/b = 0, the value of S_1 is 0.1207 and S_6 is -0.0310, and the ratio of $|S_6/S_1|$ is 0.26. Also the convergence appears to be slow. For x/b = 0.9, the value of S_1 is 1.9630 and S_6 is -0.0264, and the ratio of $|S_6/S_1|$ is 0.013. Even though the sixth term, S_6, is considerably smaller than the first term, it is larger in magnitude than S_5 by a small amount. However, for engineering purposes the value of S = 1.9259 is considered adequate. For b/h = 1.0, μ = 0.5, and x/b = 0, the ratio of $|S_6/S_1|$ is 0.35. However, observing the terms in the series, it can be seen that the convergence is more rapid than in the case where μ = 0.1. As the value of b/h decreases, the ratio of $|S_6/S_1|$ decreases rapidly. For x/b = 0, the values of $|S_6/S_1|$ are given for various parameters:

b/h	μ = 0.1	μ = 0.5
0.05	0.0002	0.0003
0.10	0.0008	0.0013
0.20	0.0034	0.0053
0.30	0.0082	0.0125
0.40	0.0155	0.0237
0.50	0.0267	0.0401
0.60	0.0430	0.0641
0.70	0.0676	0.0987
0.80	0.1043	0.1417
0.90	0.1617	0.2274
1.00	0.2568	0.3521

These ratios are the maximum possible, and they decrease rapidly as the value of x/b increases. By observing the values of S_i's for the four tabulated values of x/b, it can be seen that the convergence is slower for small values of Poisson's ratio, μ. Also the influence of the first term increases rapidly as the value of x/b increases. Thus, for practical purposes, the accuracy obtained by taking the first six terms in the series is considered quite adequate.

Table I. Stress Distribution Across the Base for b/h = 0.05

μ	x/b	S_1	S_2	S_3	S_4	S_5	S_6	S
0.1	0	7.9921	-0.0143	0.0042	-0.0023	0.0018	-0.0017	7.9798
	0.3	8.7847	-0.0143	0.0042	-0.0023	0.0018	-0.0017	8.7724
	0.6	12.4797	-0.0143	0.0042	-0.0023	0.0018	-0.0017	12.4874
	0.9	42.1545	-0.0142	0.0042	0.0023	0.0018	-0.0017	42.1425
0.25	0	7.6192	-0.0152	0.0042	-0.0020	0.0013	-0.0009	7.6065
	0.3	8.3747	-0.0152	0.0042	-0.0020	0.0013	-0.0009	8.3621
	0.6	11.9164	-0.0151	0.0042	-0.0020	0.0013	-0.0009	11.9038
	0.9	40.1873	-0.0151	0.0042	-0.0020	0.0013	-0.0009	40.1748
0.3333	0	7.6191	-0.0168	0.0048	-0.0022	0.0013	-0.0009	7.6053
	0.3	8.3747	-0.0168	0.0048	-0.0020	0.0013	-0.0009	8.3609
	0.6	11.9163	-0.0168	0.0048	-0.0020	0.0013	-0.0009	11.9025
	0.9	40.1871	-0.0168	0.0048	-0.0020	0.0013	-0.0009	40.1734
0.5	0	8.4657	-0.0275	0.0102	-0.0057	0.0038	-0.0027	8.4437
	0.3	9.3052	-0.0275	0.0102	-0.0057	0.0038	-0.0027	9.2833
	0.6	13.2404	-0.0275	0.0102	-0.0057	0.0038	-0.0027	13.2185
	0.9	44.6526	-0.0274	0.0102	-0.0057	0.0038	-0.0027	44.6307

Table II. Stress Distribution Across the Base for b/h = 0.1

μ	x/b	S_1	S_2	S_3	S_4	S_5	S_6	S
0.1	0	3.9643	-0.0284	0.0084	-0.0046	0.0035	-0.0033	3.9399
	0.3	4.3606	-0.0284	0.0084	-0.0046	0.0035	-0.0033	4.3362
	0.6	6.2183	-0.0282	0.0084	-0.0046	0.0035	-0.0033	6.1941
	0.9	21.0460	-0.0279	0.0083	-0.0046	0.0035	-0.0033	21.0220
0.25	0	3.7793	-0.0302	0.0085	-0.0040	0.0025	-0.0019	3.7542
	0.3	4.1572	-0.0301	0.0084	-0.0040	0.0025	-0.0019	4.1321
	0.6	5.9281	-0.0299	0.0084	-0.0040	0.0025	-0.0019	5.9033
	0.9	20.0638	-0.0295	0.0084	-0.0040	0.0025	-0.0019	20.0393
0.3333	0	3.7793	-0.0335	0.0096	-0.0045	0.0026	-0.0017	3.7518
	0.3	4.1571	-0.0334	0.0096	-0.0045	0.0026	-0.0017	4.1297
	0.6	5.9281	-0.0332	0.0096	-0.0045	0.0026	-0.0017	5.9009
	0.9	20.0637	-0.0328	0.0095	-0.0044	0.0026	-0.0017	20.0369
0.5	0	4.1993	-0.0547	0.0203	-0.0113	0.0075	-0.0055	4.1556
	0.3	4.6191	-0.0546	0.0203	-0.0113	0.0075	-0.0054	4.5755
	0.6	6.5868	-0.0541	0.0202	-0.0113	0.0075	-0.0054	6.5436
	0.9	22.2932	-0.0535	0.0201	-0.0113	0.0075	-0.0054	22.2505

Table III. Stress Distribution Across the Base for b/h = 0.2

μ	x/b	S_1	S_2	S_3	S_4	S_5	S_6	S
0.1	0	1.9198	-0.0556	0.0166	-0.0092	0.0070	-0.0066	1.8721
	0.3	2.1183	-0.0551	0.0166	-0.0092	0.0070	-0.0066	2.0711
	0.6	3.0483	-0.0537	0.0164	-0.0091	0.0070	-0.0066	3.0023
	0.9	10.4641	-0.0515	0.0161	-0.0090	0.0069	-0.0066	10.4200
0.25	0	1.8302	-0.0588	0.0167	-0.0080	0.0050	-0.0037	1.7814
	0.3	2.0195	-0.0583	0.0167	-0.0080	0.0050	-0.0037	1.9712
	0.6	2.9061	-0.0567	0.0165	-0.0080	0.0050	-0.0037	2.8591
	0.9	9.9757	-0.0542	0.0162	-0.0079	0.0049	-0.0037	9.9311
0.3333	0	1.8302	-0.0652	0.0190	-0.0089	0.0052	-0.0035	1.7768
	0.3	2.0195	-0.0646	0.0189	-0.0088	0.0052	-0.0035	1.9666
	0.6	2.9061	-0.0628	0.0187	-0.0088	0.0051	-0.0034	2.8548
	0.9	9.9757	-0.0599	0.0183	-0.0087	0.0051	-0.0034	9.9270
0.5	0	2.0336	-0.1064	0.0400	-0.0225	0.0149	-0.0108	1.9488
	0.3	2.2439	-0.1053	0.0399	-0.0224	0.0149	-0.0108	2.1601
	0.6	3.2290	-0.1022	0.0393	-0.0222	0.0148	-0.0108	3.1478
	0.9	11.0841	-0.0972	0.0383	-0.0218	0.0146	-0.0106	11.0074

Table IV. Stress Distribution Across the Base for b/h = 0.4

μ	x/b	s_1	s_2	s_3	s_4	s_5	s_6	s
0.1	0	0.8436	-0.1011	0.0321	-0.0180	0.0139	-0.0131	0.7522
	0.3	0.9455	-0.0981	0.0316	-0.0179	0.0138	-0.0131	0.8618
	0.6	1.4182	-0.0894	0.0305	-0.0175	0.0136	-0.0130	1.3423
	0.9	5.1380	-0.0761	0.0285	-0.0169	0.0133	-0.0128	5.0741
0.25	0	0.8042	-0.1063	0.0320	-0.0157	0.0099	-0.0073	0.7168
	0.3	0.9014	-0.1029	0.0316	-0.0155	0.0098	-0.0073	0.8170
	0.6	1.3520	-0.0932	0.0302	-0.0151	0.0096	-0.0072	1.2762
	0.9	4.8982	-0.0783	0.0280	-0.0144	0.0093	-0.0071	4.8358
0.3333	0	0.8042	-0.1175	0.0362	-0.0172	0.0101	-0.0068	0.7091
	0.3	0.9014	-0.1136	0.0356	-0.0170	0.0101	-0.0068	0.8096
	0.6	1.3520	-0.1025	0.0339	-0.0165	0.0098	-0.0067	1.2701
	0.9	4.8982	-0.0854	0.0312	-0.0157	0.0095	-0.0065	4.8313
0.5	0	0.8936	-0.1903	0.0759	-0.0433	0.0290	-0.0212	0.7436
	0.3	1.0015	-0.1837	0.0745	-0.0428	0.0287	-0.0210	0.8572
	0.6	1.5022	-0.1646	0.0704	-0.0412	0.0279	-0.0205	1.3742
	0.9	5.4425	-0.1353	0.0639	-0.0385	0.0265	-0.0197	5.3393

Table V. Stress Distribution Across the Base for b/h = 0.6

μ	x/b	S_1	S_2	S_3	S_4	S_5	S_6	S
0.1	0	0.4507	-0.1303	0.0452	-0.0261	-0.0204	-0.0194	0.3405
	0.3	0.5243	-0.1230	0.0440	-0.0257	0.0202	-0.0193	0.4205
	0.6	0.8553	-0.1026	0.0405	-0.0246	0.0196	-0.0189	0.7693
	0.9	3.3586	-0.0730	0.0351	-0.0227	0.0186	-0.0183	3.2983
0.25	0	0.4297	-0.1354	0.0448	-0.0225	0.0144	-0.0108	0.3201
	0.3	0.4998	-0.1274	0.0434	-0.0221	0.0142	-0.0107	0.3972
	0.6	0.8154	-0.1049	0.0394	-0.0208	0.0137	-0.0104	0.7322
	0.9	3.2019	-0.0724	0.0332	-0.0188	0.0128	-0.0099	3.1467
0.3333	0	0.4297	-0.1487	0.0503	-0.0246	0.0147	-0.0100	0.3114
	0.3	0.4998	-0.1396	0.0486	-0.0241	0.0145	-0.0099	0.3894
	0.6	0.8154	-0.1141	0.0438	-0.0225	0.0138	-0.0095	0.7268
	0.9	3.2018	-0.0773	0.0363	-0.0200	0.0127	-0.0089	3.1446
0.5	0	0.4775	-0.2381	0.1041	-0.0612	0.0415	-0.0306	0.2932
	0.3	0.5554	-0.2226	0.1001	-0.0595	0.0406	-0.0300	0.3839
	0.6	0.9060	-0.1794	0.0886	-0.0546	0.0380	-0.0288	0.7700
	0.9	3.5576	-0.1173	0.0709	-0.0469	0.0338	-0.0258	3.4723

Table VI. Stress Distribution Across the Base for b/h = 0.8

μ	x/b	S_1	S_2	S_3	S_4	S_5	S_6	S
0.1	0	0.2434	-0.1416	0.0554	-0.0333	0.0264	-0.0254	0.1249
	0.3	0.3057	-0.1308	0.0530	-0.0324	0.0259	-0.0251	0.1964
	0.6	0.5740	-0.1005	0.0462	-0.0299	0.0246	-0.0243	0.4902
	0.9	2.4800	-0.0574	0.0360	-0.0260	0.0226	-0.0229	2.4323
0.25	0	0.2320	-0.1448	0.0541	-0.0283	0.0185	-0.0140	0.1174
	0.3	0.2915	-0.1332	0.0514	-0.0274	0.0180	-0.0138	0.1865
	0.6	0.5472	-0.1006	0.0438	-0.0248	0.0169	-0.0131	0.4694
	0.9	2.3643	-0.0543	0.0326	-0.0208	0.0150	-0.0121	2.3246
0.3333	0	0.2320	-0.1575	0.0602	-0.0307	0.0187	-0.0128	0.1099
	0.3	0.2915	-0.1445	0.0570	-0.0296	0.0182	-0.0126	0.1800
	0.6	0.5472	-0.1080	0.0478	-0.0263	0.0167	-0.0118	0.4657
	0.9	2.3643	-0.0563	0.0344	-0.0214	0.0145	-0.0106	2.3249
0.5	0	0.2578	-0.2478	0.1224	-0.0749	0.0518	-0.0386	0.0707
	0.3	0.3239	-0.2262	0.1148	-0.0714	0.0499	-0.0374	0.1535
	0.6	0.6081	-0.1657	0.0936	-0.0616	0.0444	-0.0340	0.4848
	0.9	2.6270	-0.0801	0.0627	-0.0468	0.0359	-0.0286	2.5702

Table VII. Stress Distribution Across the Base for b/h = 1.0

μ	x/b	S_1	S_2	S_3	S_4	S_5	S_6	S
0.1	0	0.1207	-0.1378	0.0623	-0.0392	0.0318	-0.0310	0.0068
	0.3	0.1771	-0.1263	0.0586	-0.0377	0.0310	-0.0305	0.0721
	0.6	0.4106	-0.0924	0.0481	-0.0335	0.0287	-0.0289	0.3325
	0.9	1.9630	-0.0418	0.0330	-0.0271	0.0251	-0.0264	1.9259
0.25	0	0.1150	-0.1380	0.0597	-0.0329	0.0220	-0.0159	0.0089
	0.3	0.1688	-0.1261	0.0556	-0.0314	0.0212	-0.0165	0.0717
	0.6	0.3914	-0.0909	0.0443	-0.0270	0.0191	-0.0153	0.3217
	0.9	1.8714	-0.0377	0.0282	-0.0205	0.0159	-0.0135	1.8438
0.3333	0	0.1150	-0.1481	0.0656	-0.0352	0.0220	-0.0153	0.0040
	0.3	0.1688	-0.1352	0.0608	-0.0333	0.0211	-0.0149	0.0673
	0.6	0.3914	-0.0965	0.0474	-0.0281	0.0186	-0.0135	0.3194
	0.9	1.8714	-0.0378	0.0285	-0.0203	-0.0148	0.0114	1.8452
0.5	0	0.1278	-0.2274	0.1302	-0.0838	0.0594	-0.0450	-0.0387
	0.3	0.1876	-0.2068	0.1193	-0.0783	0.0562	-0.0429	0.0351
	0.6	0.4349	-0.1448	0.0892	-0.0628	0.0471	-0.0370	0.3266
	0.9	2.0794	-0.0500	0.0470	-0.0403	0.0335	-0.0280	2.0416

Appendix B

A_i SERIES, EQUATION 27

I. EXPRESSIONS FOR A_i's

The integrations of Equation 27 become less involved if they are first performed with respect to x and then with respect to α, starting from Equation 21. The expressions for A_i's are given by Equation 32. The following notations are used in Equation 32:

$$C(i)(J) = \left[\frac{b}{ih}\left(1 + \frac{x_{cr}}{b}\right)\right]^J$$

$$D(i)(J) = \left[\frac{b}{ih}\left(1 - \frac{x_{cr}}{b}\right)\right]^J$$

$$C(i)2(J) = \left\{1 + \left[\frac{b}{ih}\left(1 + \frac{x_{cr}}{b}\right)\right]^2\right\}^J \tag{31}$$

$$D(i)2(J) = \left\{1 + \left[\frac{b}{ih}\left(1 - \frac{x_{cr}}{b}\right)\right]^2\right\}^J$$

where $i \geq j$ and the last two notations are used only in the denominators except when used with logarithms.

$$A_1 = \frac{P_y^{(1)}}{pb}$$

$$= \frac{C_4}{\left(\frac{b}{h}\right)\left(\frac{ph}{dE}\right)}\left[\frac{1}{2}\log\frac{C421}{D421} + 2\left\{-\frac{1}{C221} + \frac{1}{D221}\right\} - \log\frac{C11}{D11}\right] \tag{32a}$$

$$A_2 = \frac{P_y^{(2)}}{pb}$$

$$= \frac{C_4}{\left(\frac{b}{h}\right)\left(\frac{ph}{dE}\right)}\left[C_1 \left\{ - 0.1111111 \left[\frac{-1 + C62}{C622} - \frac{-1 + D62}{D622} \right] \right.\right.$$

$$- 0.05 \left[\frac{-1 + 3(C42)}{C423} - \frac{-1 + 3(D42)}{D423} \right]$$

$$\left. + 1.0 \left[\frac{-1 + C22}{C222} - \frac{-1 + D22}{D222} \right] \right\}$$

$$\left. + C_2 \left\{ - 0.5 \log \frac{C621}{D621} - \left[- \frac{1}{C421} + \frac{1}{D421} \right] + 0.5 \log \frac{C221}{D221} \right\} \right] \qquad (32b)$$

$$A_3 = \frac{P_y^{(3)}}{pb} = \frac{C_4}{\left(\frac{b}{h}\right)\left(\frac{ph}{dE}\right)}\Bigg[$$

$$C_1^2 \left\{ 0.0234375 \left[\frac{-1 + 6(C82) - C84}{C824} - \frac{-1 + 6(D82) - D84}{D824} \right] \right.$$

$$+ 0.1975309 \left[\frac{-1 + 10(C62) - 5(C64)}{C625} - \frac{-1 + 10(D62) - 5(D64)}{D625} \right]$$

$$\left. - 0.375 \left[\frac{-1 + 6(C42) - C44}{C424} - \frac{-1 + 6(D42) - D44}{D424} \right] \right\}$$

$$+ C_1 C_2 \left\{ 0.125 \left[\frac{-1 + C82}{C822} - \frac{-1 + D82}{D822} \right] \right.$$

$$+ 0.2962963 \left[\frac{-1 + 3(C62)}{C623} - \frac{-1 + 3(D62)}{D623} \right]$$

$$- 0.50 \left[\frac{-1 + C42}{C422} - \frac{-1 + D42}{D422} \right] \Bigg\}$$

$$+ (C_2{}^2 - 1) \left\{ 0.5 \log \frac{C821}{D821} + 0.6666667 \left[- \frac{1}{C621} + \frac{1}{D621} \right] - 0.5 \log \frac{C421}{D421} \right\} \Bigg] \qquad (32c)$$

$$A_4 = \frac{P_y^{(4)}}{pb} = \frac{C_4}{\left(\frac{b}{h}\right)\left(\frac{ph}{dE}\right)} \Bigg[$$

$$C_1{}^3 \Bigg\{ - 0.00768 \left[\frac{-1 + 15(C102) - 15(C104) + C106}{C1026} - \frac{-1 + 15(D102) - 15(D104) + D106}{D1026} \right]$$

$$- 0.0878906 \left[\frac{-1 + 21(C82) - 35(C84) + 7(C86)}{C827} - \frac{-1 + 21(D82) - 35(D84) + 7(D86)}{D827} \right]$$

$$+ 0.1646091 \left[\frac{-1 + 15(C62) - 15(C64) + C66}{C626} - \frac{-1 + 15(D62) - 15(D64) + D66}{D626} \right] \Bigg\}$$

$$+ C_1{}^2 C_2 \Bigg\{ - 0.0288 \left[\frac{-1 + 6(C102) - C104}{C1024} - \frac{-1 + 6(D102) - D104}{D1024} \right]$$

$$- 0.140625 \left[\frac{-1 + 10(C82) - 5(C84)}{C825} - \frac{-1 + 10(D82) - 5(D84)}{D825} \right]$$

$$+ 0.2222222 \left[\frac{-1 + 6(C62) - C64}{C624} - \frac{-1 + 6(D62) - D64}{D624} \right] \Bigg\}$$

$$+ (3C_1C_2^{\,2} - 2C_1) \Bigg\{ - 0.04 \left[\frac{-1 + C102}{C1022} - \frac{-1 + D102}{D1022} \right]$$

$$- 0.0625 \left[\frac{-1 + 3(C82)}{C823} - \frac{-1 + 3(D82)}{D823} \right]$$

$$+ 0.1111111 \left[\frac{-1 + C62}{C622} - \frac{-1 + D62}{D622} \right] \Bigg\}$$

$$+ (C_2^{\,3} - 2C_2) \Bigg\{ - 0.5 \log \frac{C1021}{D1021} - 0.5 \left[- \frac{1}{C821} + \frac{1}{D821} \right] + 0.5 \log \frac{C621}{D621} \Bigg\} \Bigg] \qquad (32d)$$

$$A_5 = \frac{P_y^{(5)}}{pb} = \frac{C_4}{\left(\frac{b}{h}\right)\left(\frac{ph}{dE}\right)} \Bigg[$$

$$C_1^{\,4} \Bigg\{ 0.0030007 \left[\frac{-1 + 28(C122) - 70(C124) + 28(C126) - C128}{C1228} \right.$$

$$\left. - \frac{-1 + 28(D122) - 70(D124) + 28(D126) - D128}{D1228} \right]$$

$$+ 0.0412877 \left[\frac{-1 + 36(C102) - 126(C104) + 84(C106) - 9(C108)}{C1029} \right.$$

$$\left. - \frac{-1 + 36(D102) - 126(D104) + 84(D106) - 9(D108)}{D1029} \right]$$

$$- 0.0769043 \left[\frac{-1 + 28(C82) - 70(C84) + 28(C86) - C88}{C828} \right.$$

$$\left. \left. - \frac{-1 + 28(D82) - 70(D84) + 28(D86) - D88}{D828} \right] \right\}$$

$$+ C_1^{\,3} C_2 \left\{ 0.0102881 \left[\frac{-1 + 15(C122) - 15(C124) + C126}{C1226} - \frac{-1 + 15(D122) - 15(D124) + D126}{D1226} \right] \right.$$

$$+ 0.0737280 \left[\frac{-1 + 21(C102) - 35(C104) + 7(C106)}{C1027} - \frac{-1 + 21(D102) - 35(D104) + 7(D106)}{D1027} \right]$$

$$\left. - 0.1171875 \left[\frac{-1 + 15(C82) - 15(C84) + C86}{C826} - \frac{-1 + 15(D82) - 15(D84) + D86}{D826} \right] \right\}$$

$$+ (2C_2^{\,2} - 1)\, C_1^{\,2} \left\{ 0.0138889 \left[\frac{-1 + 6(C122) - C124}{C1224} - \frac{-1 + 6(D122) - D124}{D1224} \right] \right.$$

$$+ 0.04608 \left[\frac{-1 + 10(C102) - 5(C104)}{C1025} - \frac{-1 + 10(D102) - 5(D104)}{D1025} \right]$$

$$- 0.0703125 \left[\frac{-1 + 6(C82) - C84}{C824} - \frac{-1 + 6(D82) - D84}{D824} \right] \Bigg\}$$

$$+ (2C_2^{\,2} - 3)\, C_1 C_2 \Bigg\{ 0.0555556 \left[\frac{-1 + C122}{C1222} - \frac{-1 + D122}{D1222} \right]$$

$$+ 0.064 \left[\frac{-1 + 3(C102)}{C1023} - \frac{-1 + 3(D102)}{D1023} \right]$$

$$- 0.125 \left[\frac{-1 + C82}{C822} - \frac{-1 + D82}{D822} \right] \Bigg\}$$

$$+ (C_2^{\,4} - 3C_2^{\,2} + 1) \Bigg\{ 0.5 \log \frac{C1221}{D1221} + 0.4 \left[-\frac{1}{C1021} + \frac{1}{D1021} \right] - 0.5 \log \frac{C821}{D821} \Bigg\} \Bigg] \qquad (32e)$$

$$A_6 = \frac{P_y^{(6)}}{pb} = \frac{C_4}{\left(\frac{b}{h}\right)\left(\frac{ph}{dE}\right)} \Bigg[$$

$$C_1^{\,5} \Bigg\{ - 0.0012846 \left[\frac{-1 + 45(C142) - 210(C144) + 210(C146) - 45(C148) + C1410}{C14210} \right.$$

$$- \frac{-1 + 45(D142) - 210(D144) + 210(D146) - 45(D148) + D1410}{D14210} \Bigg]$$

$$- 0.0200046 \Bigg[\frac{-1 + 55(C122) - 330(C124) + 462(C126) - 165(C128) + 11(C1210)}{C12211}$$

$$- \frac{-1 + 55(D122) - 330(D124) + 462(D126) - 165(D128) + 11(D1210)}{D12211} \Bigg]$$

$$+ 0.0371589 \Bigg[\frac{-1 + 45(C102) - 210(C104) + 210(C106) - 45(C108) + C1010}{C10210}$$

$$- \frac{-1 + 45(D102) - 210(D104) + 210(D106) - 45(D108) + D1010}{D10210} \Bigg] \Bigg\}$$

$$+ C_1{}^4C_2 \Bigg\{ - 0.0043714 \Bigg[\frac{-1 + 28(C142) - 70(C144) + 28(C146) - C148}{C1428}$$

$$- \frac{-1 + 28(D142) - 70(D144) + 28(D146) - D148}{D1428} \Bigg]$$

$$- 0.0400091 \Bigg[\frac{-1 + 36(C122) - 126(C124) + 84(C126) - 9(C128)}{C1229}$$

$$- \frac{-1 + 36(D122) - 126(D124) + 84(D126) - 9(D128)}{D1229} \Bigg]$$

$$+ 0.064512 \Bigg[\frac{-1 + 28(C102) - 70(C104) + 28(C106) - C108}{C1028}$$

$$- \frac{-1 + 28(D102) - 70(D104) + 28(D106) - D108}{D1028} \Bigg] \Bigg\}$$

$$+ C_1^{\,3} \, (5C_2^{\,2} - 2) \Bigg\{ - 0.002040 \Bigg[\frac{-1 + 15(C142) - 15(C144) + C146}{C1426}$$

$$- \frac{-1 + 15(D142) - 15(D144) + D146}{D1426} \Bigg]$$

$$- 0.0102881 \Bigg[\frac{-1 + 21(C122) - 35(C124) + 7(C126)}{C1227}$$

$$- \frac{-1 + 21(D122) - 35(D124) + 7(D126)}{D1227} \Bigg]$$

$$+ 0.015360 \Bigg[\frac{-1 + 15(C102) - 15(C104) + C106}{C1026}$$

$$- \frac{-1 + 15(D102) - 15(D104) + D106}{D1026} \Bigg] \Bigg\}$$

$$+ C_1^{\,2} (5C_2^{\,3} - 6C_2) \Bigg\{ - 0.0049979 \left[\frac{-1 + 6(C142) - C144}{C1424} - \frac{-1 + 6(D142) - D144}{D1424} \right]$$

$$- 0.0123457 \left[\frac{-1 + 10(C122) - 5(C124)}{C1225} - \frac{-1 + 10(D122) - 5(D124)}{D1225} \right]$$

$$+ 0.0192 \left[\frac{-1 + 6(C102) - C104}{C1024} - \frac{-1 + 6(D102) - D104}{D1024} \right] \Bigg\}$$

$$+ C_1 (5C_2^{\,4} - 12C_2^{\,2} + 3) \Bigg\{ - 0.0204082 \left[\frac{-1 + C142}{C1422} - \frac{-1 + D142}{D1422} \right]$$

$$- 0.0185185 \left[\frac{-1 + 3(C122)}{C1223} - \frac{-1 + 3(D122)}{D1223} \right]$$

$$+ 0.04 \left[\frac{-1 + C102}{C1022} - \frac{-1 + D102}{D1022} \right] \Bigg\}$$

$$+ (C_2^{\,5} - 4C_2^{\,3} + 3C_2) \Bigg\{ - 0.5 \log \frac{C1421}{D1421} - 0.3333333 \left[- \frac{1}{C1221} + \frac{1}{D1221} \right] + 0.5 \log \frac{C1021}{D1021} \Bigg\} \Bigg] \quad (32f)$$

II. ACCURACY OF THE SOLUTIONS

Typical results are given in Tables VIII through XIV for the digital computer evaluation of Equation 28 for the first six A_i terms, considering the parameters b/h, μ, ph/dE, and x_{cr}/b. As for the S_i's, the convergence of the series is very difficult to prove. It is presumed that the results are acceptable if the last term of the series is sufficiently small compared to the first term. From the results it can be observed that the error is large for large values of b/h, small values of ph/dE, and large values of μ. However, the convergence is poor for small values of μ. The following table represents the maximum ratio of $|A_6/A_1|$ for all values of b/h where $\mu = 0.5$.

b/h	ph/dE	Values of $\lvert A_6/A_1 \rvert$
0.05	9.0	0.0004
0.1	5.0	0.0011
0.2	2.0	0.0051
0.3	1.25	0.0121
0.4	0.9	0.0223
0.5	0.5	0.0395
0.6	0.3	0.0635
0.7	0.2	0.0956
0.8	0.06	0.1465
0.9	0.01	0.2260
1.0	0.01	0.3166

For other values of ph/dE and μ the error is smaller than that indicated above.

Table VIII. Arching Due to Deflection of Strip for b/h = 0.05

μ	ph/dE	x_{cr}/b	A_1	A_2	A_3	A_4	A_5	A_6	A	Arching (%)
0.1	0.01	0								100
	0.1	0								100
	1	0								100
	10	0.4487	0.3862	-0.0006	0.0002	-0.0001	0.0001	-0.0001	0.3856	93.69
	100	0.9591	0.1548	-0.0001	0.0000	-0.0000	0.0000	-0.0000	0.1547	19.56
	1000	0.9960	0.0249	-0.0000	0.0000	-0.0000	0.0000	-0.0000	0.0248	2.89
0.25	0.01	0								100
	0.1	0								100
	1	0								100
	10	0.4884	0.4069	-0.0007	0.0002	-0.0001	0.0001	-0.0000	0.4063	91.79
	100	0.9611	0.1495	-0.0001	0.0000	-0.0000	0.0000	-0.0000	0.1494	18.83
	1000	0.9962	0.0239	-0.0000	0.0000	-0.0000	0.0000	-0.0000	0.0239	2.77
0.3333	0.01	0								100
	0.1	0								100
	1	0								100
	10	0.4885	0.4070	-0.0008	0.0002	-0.0001	0.0001	-0.0000	0.4064	91.78
	100	0.9611	0.1495	-0.0002	0.0000	-0.0000	0.0000	-0.0000	0.1494	18.83
	1000	0.9962	0.0239	-0.0000	0.0000	-0.0000	0.0000	-0.0000	0.0239	2.77
0.5	0.01	0								100
	0.1	0								100
	1	0								100
	10	0.3936	0.3523	-0.0011	0.0004	-0.0002	0.0001	-0.0001	0.3514	95.78
	100	0.9566	0.1615	-0.0003	0.0001	-0.0001	0.0000	0.0000	0.1612	20.46
	1000	0.9957	0.0261	-0.0000	0.0000	-0.0000	0.0000	0.0000	0.0261	3.03

Table IX. Arching Due to Deflection of Strip for b/h = 0.1

μ	ph/dE	x_{cr}/b	A_1	A_2	A_3	A_4	A_5	A_6	A	Arching (%)
0.1	0.01	0								100
	0.1	0								100
	1	0								100
	10	0.7759	0.4114	-0.0022	0.0006	-0.0004	0.0003	-0.0003	0.4095	63.36
	100	0.9798	0.0914	-0.0003	0.0001	-0.0000	0.0000	-0.0000	0.0912	11.14
	1000	0.9980	0.0138	-0.0000	0.0000	-0.0000	0.0000	-0.0000	0.0138	1.58
0.25	0.01	0								100
	0.1	0								100
	1	0								100
	10	0.7877	0.4038	-0.0024	0.0007	-0.0003	0.0002	-0.0001	0.4018	61.41
	100	0.9807	0.0881	-0.0003	0.0001	-0.0000	0.0000	-0.0000	0.0878	10.71
	1000	0.9981	0.0132	-0.0000	0.0000	-0.0000	0.0000	-0.0000	0.0132	1.51
0.3333	0.01	0								100
	0.1	0								100
	1	0								100
	10	0.7878	0.4038	-0.0026	0.0006	-0.0004	0.0002	-0.0001	0.4017	61.39
	100	0.9807	0.0881	-0.0003	0.0001	-0.0000	0.0000	-0.0000	0.0878	10.71
	1000	0.9981	0.0132	-0.0000	0.0000	-0.0000	0.0000	-0.0000	0.0132	1.51
0.5	0.01	0								100
	0.1	0								100
	1	0								100
	10	0.7611	0.4205	-0.0041	0.0015	-0.0009	0.0006	-0.0004	0.4172	65.61
	100	0.9786	0.0956	-0.0005	0.0002	-0.0001	0.0001	-0.0001	0.0952	11.66
	1000	0.9979	0.0145	-0.0001	0.0000	-0.0000	0.0000	-0.0000	0.0144	1.66

Table X. Arching Due to Deflection of Strip for b/h = 0.2

μ	ph/dE	x_{cr}/b	A_1	A_2	A_3	A_4	A_5	A_6	A	Arching (%)
0.1	0.01	0								100
	0.1	0								100
	1	0								100
	10	0.8956	0.2830	-0.0049	0.0015	-0.0008	0.0006	-0.0006	0.2789	38.33
	100	0.9899	0.0522	-0.0005	0.0002	-0.0001	0.0001	-0.0001	0.0517	6.18
	1000	0.9990	0.0075	-0.0001	0.0000	0.0000	0.0000	-0.0000	0.0075	0.85
0.25	0.01	0								100
	0.1	0								100
	1	0								100
	10	0.9007	0.2748	-0.0052	0.0015	-0.0007	0.0004	-0.0003	0.2706	36.98
	100	0.9904	0.0502	-0.0006	0.0002	-0.0001	0.0000	-0.0000	0.0497	5.93
	1000	0.9990	0.0072	-0.0001	0.0000	-0.0000	0.0000	-0.0000	0.0072	0.81
0.3333	0.01	0								100
	0.1	0								100
	1	0								100
	10	0.9008	0.2749	-0.0057	0.0017	-0.0008	0.0005	-0.0003	0.2702	36.94
	100	0.9904	0.0502	-0.0006	0.0002	-0.0001	0.0001	-0.0000	0.0497	5.93
	1000	0.9990	0.0072	-0.0001	0.0000	0.0000	0.0000	-0.0000	0.0072	0.81
0.5	0.01	0								100
	0.1	0								100
	1	0								100
	10	0.8895	0.2935	-0.0092	0.0035	-0.0020	0.0013	-0.0010	0.2861	39.67
	100	0.9893	0.0546	-0.0010	0.0004	-0.0002	0.0001	-0.0001	0.0538	6.45
	1000	0.9989	0.0079	-0.0001	0.0000	-0.0000	0.0000	-0.0000	0.0078	0.89

Table XI. Arching Due to Deflection of Strip for b/h = 0.4

μ	ph/dE	x_{cr}/b	A_1	A_2	A_3	A_4	A_5	A_6	A	Arching (%)
0.1	0.01	0								100
	0.1	0								100
	1	0.4331	0.3969	-0.0429	0.0138	-0.0078	0.0060	-0.0057	0.3603	92.72
	10	0.9496	0.1689	-0.0087	0.0029	-0.0017	0.0013	-0.0012	0.1615	21.19
	100	0.9950	0.0285	-0.0009	0.0003	-0.0002	0.0001	-0.0001	0.0277	3.28
	1000	More than 100 iterations to find x_{cr}								
0.25	0.01	0								100
	0.1	0								100
	1	0.4691	0.4163	-0.0486	0.0149	-0.0073	0.0046	-0.0034	0.3764	90.74
	10	0.9520	0.1634	-0.0091	0.0029	-0.0014	0.0009	-0.0007	0.1560	20.40
	100	0.9952	0.0274	-0.0009	0.0003	-0.0002	0.0001	-0.0001	0.0266	3.14
	1000	0.9995	0.0038	-0.0001	0.0000	-0.0000	0.0000	-0.0000	0.0038	0.42
0.3333	0.01	0								100
	0.1	0								100
	1	0.4734	0.4210	-0.0541	0.0169	-0.0081	0.0046	-0.0032	0.3773	90.39
	10	0.9520	0.1634	-0.0100	0.0033	-0.0016	0.0009	-0.0006	0.1554	20.34
	100	0.9952	0.0274	-0.0010	0.0003	-0.0002	0.0001	-0.0001	0.0266	3.14
	1000	More than 100 iterations to find x_{cr}								
0.5	0.01	0								100
	0.1	0								100
	1	0.4286	0.4153	-0.0796	0.0321	-0.0184	0.0123	-0.0090	0.3527	92.40
	10	0.9468	0.1760	-0.0160	0.0068	-0.0039	0.0027	-0.0020	0.1635	21.67
	100	0.9947	0.0299	-0.0017	0.0007	-0.0004	0.0003	-0.0002	0.0286	3.39
	1000	0.9995	0.0042	-0.0002	0.0001	-0.0000	0.0000	-0.0000	0.0041	0.46

Table XII. Arching Due to Deflection of Strip for b/h = 0.6

μ	ph/dE	x_{cr}/b	A_1	A_2	A_3	A_4	A_5	A_6	A	Arching (%)
0.1	0.01	0								100
	0.1	0								100
	1	0.6853	0.4203	-0.0809	0.0296	-0.0174	0.0137	-0.0132	0.3521	66.68
	10	0.9667	0.1176	-0.0104	0.0040	-0.0024	0.0019	-0.0018	0.1089	14.21
	100	0.9967	0.0194	-0.0011	0.0004	-0.0002	0.0002	-0.0002	0.0186	2.19
	1000	More than 100 iterations to find x_{cr}								
0.25	0.01	0								100
	0.1	0								100
	1	0.6990	0.4150	-0.0849	0.0296	-0.0152	0.0098	-0.0074	0.3469	64.79
	10	0.9683	0.1136	-0.0106	0.0039	-0.0020	+0.0013	-0.0010	0.1051	13.69
	100	0.9968	0.0187	-0.0011	0.0004	-0.0002	0.0001	-0.0001	0.0178	2.10
	1000	0.9997	0.0026	-0.0001	0.0000	-0.0000	+0.0000	-0.0000	0.0025	0.28
0.3333	0.01	0								100
	0.1	0								100
	1	0.7002	0.4164	-0.0930	0.0331	-0.0166	0.0100	-0.0068	0.3431	64.29
	10	0.9683	0.1136	-0.0116	0.0043	-0.0022	0.0013	-0.0009	0.1045	13.63
	100	0.9968	0.0187	-0.0012	0.0004	-0.0002	0.0001	-0.0001	0.0178	2.09
	1000	More than 100 iterations to find x_{cr}								
0.5	0.01	0								100
	0.1	0								100
	1	0.6797	0.4388	-0.1445	0.0662	-0.0397	0.0272	-0.0202	0.3279	64.82
	10	0.9648	0.1226	-0.0183	0.0088	-0.0054	0.0037	-0.0028	0.1086	14.38
	100	0.9965	0.0204	-0.0019	0.0009	-0.0006	0.0004	-0.0003	0.0190	2.25
	1000	More than 100 iterations to find x_{cr}								

Table XIII. Arching Due to Deflection of Strip for b/h = 0.8

μ	ph/dE	x_{cr}/b	A_1	A_2	A_3	A_4	A_5	A_6	A	Arching (%)
0.1	0.01	0								100
	0.1	0								100
	1	0.7728	0.3364	-0.0919	0.0389	-0.0243	0.0196	-0.0191	0.2597	48.69
	10	0.9751	0.0885	-0.0104	0.0046	-0.0030	0.0024	-0.0024	0.0797	10.47
	100	0.9975	0.0146	-0.0011	0.0005	-0.0003	0.0002	-0.0002	0.0137	1.62
	1000	0.9997	0.0020	-0.0001	0.0000	-0.0000	0.0000	-0.0000	0.0019	0.22
0.25	0.01	0								100
	0.1	0								100
	1	0.7813	0.3295	-0.0937	0.0377	-0.0206	0.0137	-0.0106	0.2561	47.49
	10	0.9762	0.0854	-0.0105	0.0044	-0.0025	0.0017	-0.0013	0.0773	10.10
	100	0.9976	0.0140	-0.0011	0.0004	-0.0003	0.0002	-0.0001	0.0132	1.56
	1000	More than 100 iterations to find x_{cr}								
0.3333	0.01	0								100
	0.1	0								100
	1	0.7815	0.3298	-0.1012	0.0416	-0.0221	0.0137	-0.0096	0.2522	47.07
	10	0.9762	0.0854	-0.0113	0.0048	-0.0026	0.0017	-0.0012	0.0769	10.06
	100	0.9976	0.0140	-0.0011	0.0005	-0.0003	0.0002	-0.0001	0.0132	1.56
	1000	0.9998	0.0020	-0.0001	0.0000	-0.0000	0.0000	-0.0000	0.0019	0.21
0.5	0.01	0								100
	0.1	0.1816	0.4821	-0.4453	0.2206	-0.1352	0.0936	-0.0699	0.1460	96.43
	1	0.7652	0.3479	-0.1555	0.0817	-0.0518	0.0366	-0.0276	0.2313	46.61
	10	0.9736	0.0922	-0.0174	0.0095	-0.0062	0.0044	-0.0034	0.0792	10.56
	100	0.9973	0.0153	-0.0018	0.0010	-0.0006	0.0005	-0.0003	0.0140	1.67
	1000	More than 100 iterations to find x_{cr}								

Table X[illegible]. Arching Due to Deflection of Strip for b/h = 1.0

μ	ph/dE	x_{cr}/b	A_1	A_2	A_3	A_4	A_5	A_6	A	Arching (%)
0.1	0.01	0.0686	0.8342	-0.9441	0.4270	0.2688	0.2179	-0.2125	0.0537	98.51
	0.1	0.3541	0.5191	-0.4691	0.2145	-0.1364	0.1112	-0.1188	0.1305	77.64
	1	0.8187	0.2652	-0.0899	0.0439	-0.0292	0.0245	-0.0243	0.1900	37.14
	10	0.9801	0.0701	-0.0097	0.0049	-0.0034	0.0029	-0.0029	0.0620	8.19
	100	0.9980	0.0116	-0.0010	0.0005	-0.0003	0.0003	-0.0003	0.0108	1.28
	1000			More than 100 iterations to find x_{cr}						
0.25	0.01	0.0405	0.4675	-0.5589	0.2418	-0.1332	0.0889	-0.0685	0.0376	99.70
	0.1	0.3569	0.5004	-0.4725	0.2062	-0.1148	0.0771	-0.0597	0.1367	77.98
	1	0.8250	0.2592	-0.0895	0.0413	-0.0241	0.0167	-0.0131	0.1904	36.55
	10	0.9810	0.0677	-0.0095	0.0046	-0.0027	0.0019	-0.0015	0.0604	7.94
	100	0.9981	0.0112	-0.0010	0.0005	-0.0003	0.0002	-0.0002	0.0104	1.23
	1000	0.9998	0.0016	-0.0001	+0.0000	-0.0000	0.0000	-0.0000	0.0015	.17
0.3333	0.01	0.0948	1.1072	-1.4010	0.6208	-0.3331	0.2082	-0.1454	0.0566	96.18
	0.1	0.3641	0.5144	-0.5163	0.2303	-0.1248	0.0785	-0.0551	0.1271	76.30
	1	0.8249	0.2592	-0.0956	0.0448	-0.0253	0.0164	-0.0117	0.1877	36.28
	10	0.9810	0.0677	-0.0101	0.0049	-0.0029	0.0019	-0.0013	0.0602	7.92
	100	0.9981	0.0112	-0.0010	0.0005	-0.0003	0.0002	-0.0001	0.0104	1.23
	1000			More than 100 iterations to find x_{cr}						
0.5	0.01	0.2461	3.4632	-5.4825	3.1421	-2.0309	1.4450	-1.0963	-0.5593	19.46
	0.1	0.4017	0.6578	-0.8639	0.4967	-0.3233	0.2311	-0.1758	0.0225	62.08
	1	0.8107	0.2723	-0.1438	0.0854	-0.0576	0.0421	-0.0326	0.1658	35.51
	10	0.9789	0.0730	-0.0153	0.0093	-0.0064	0.0048	-0.0037	0.0617	8.28
	100	0.9979	0.0122	-0.0015	0.0009	-0.0007	0.0005	-0.0004	0.0110	1.32
	1000			More than 100 iterations to find x_{cr}						

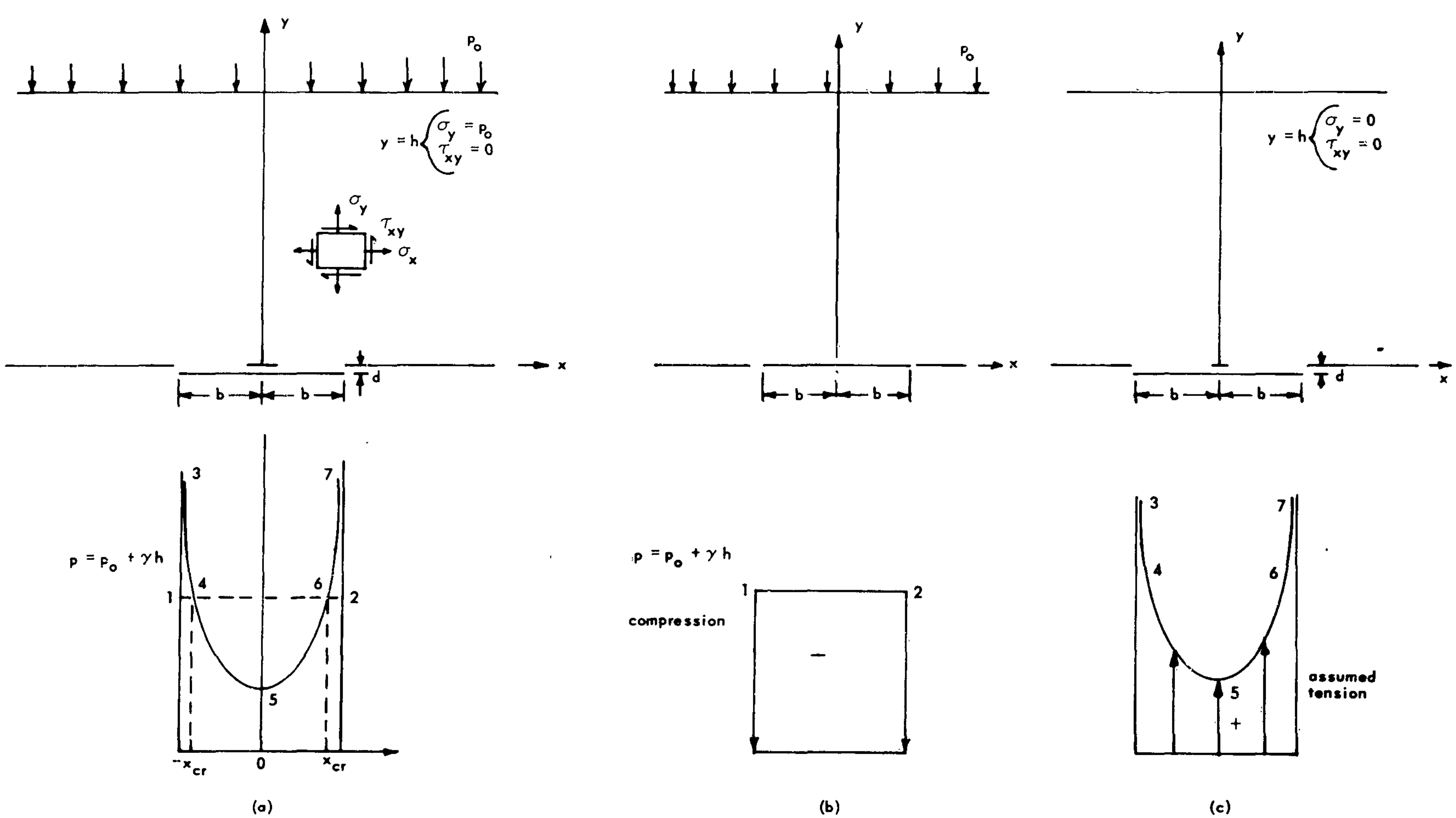

Figure 1. Stress distributions across the width of a rigid horizontal strip in an elastic medium.

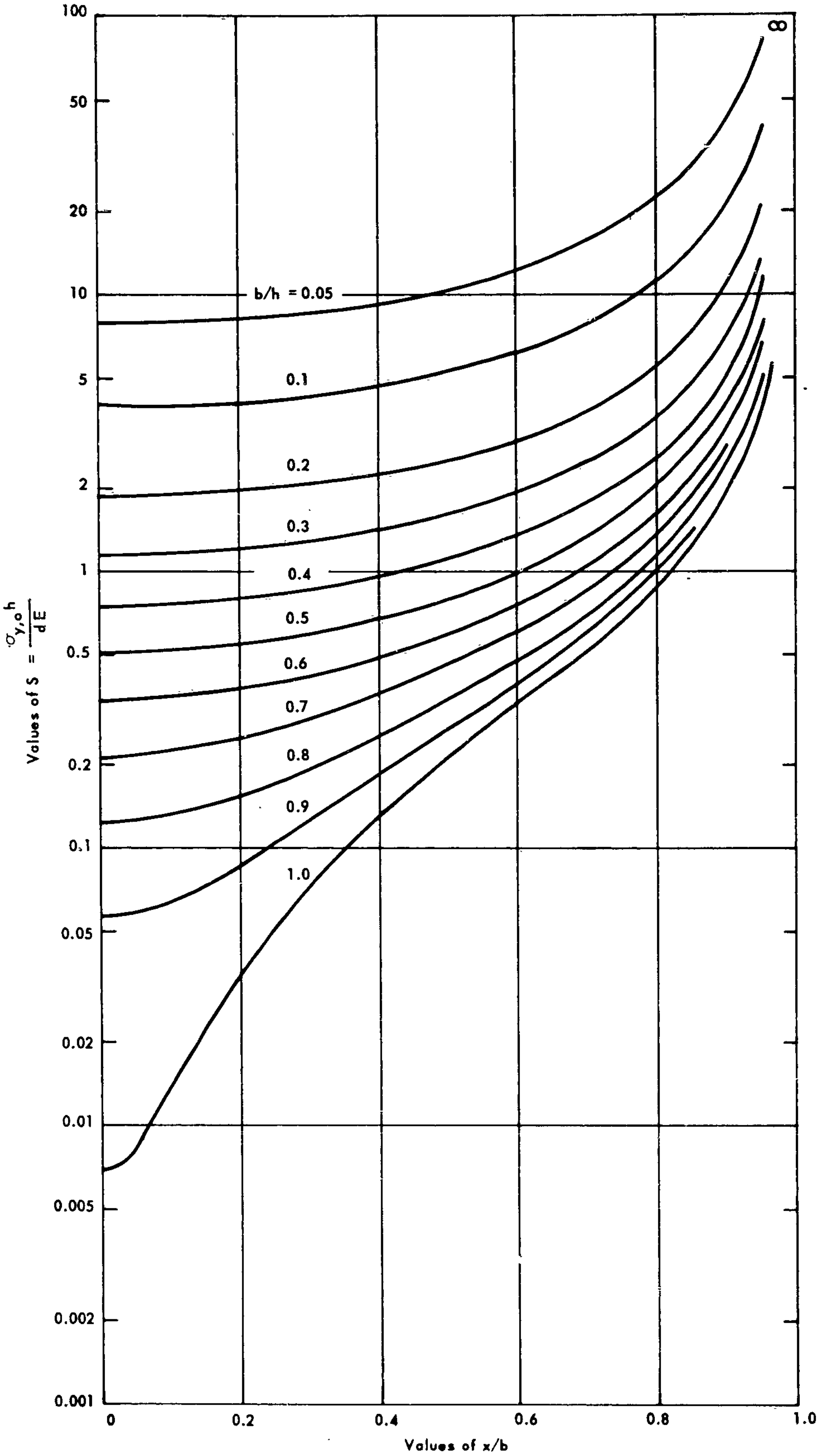

Figure 2. Pressure distribution on the base for $\mu = 0.1$.

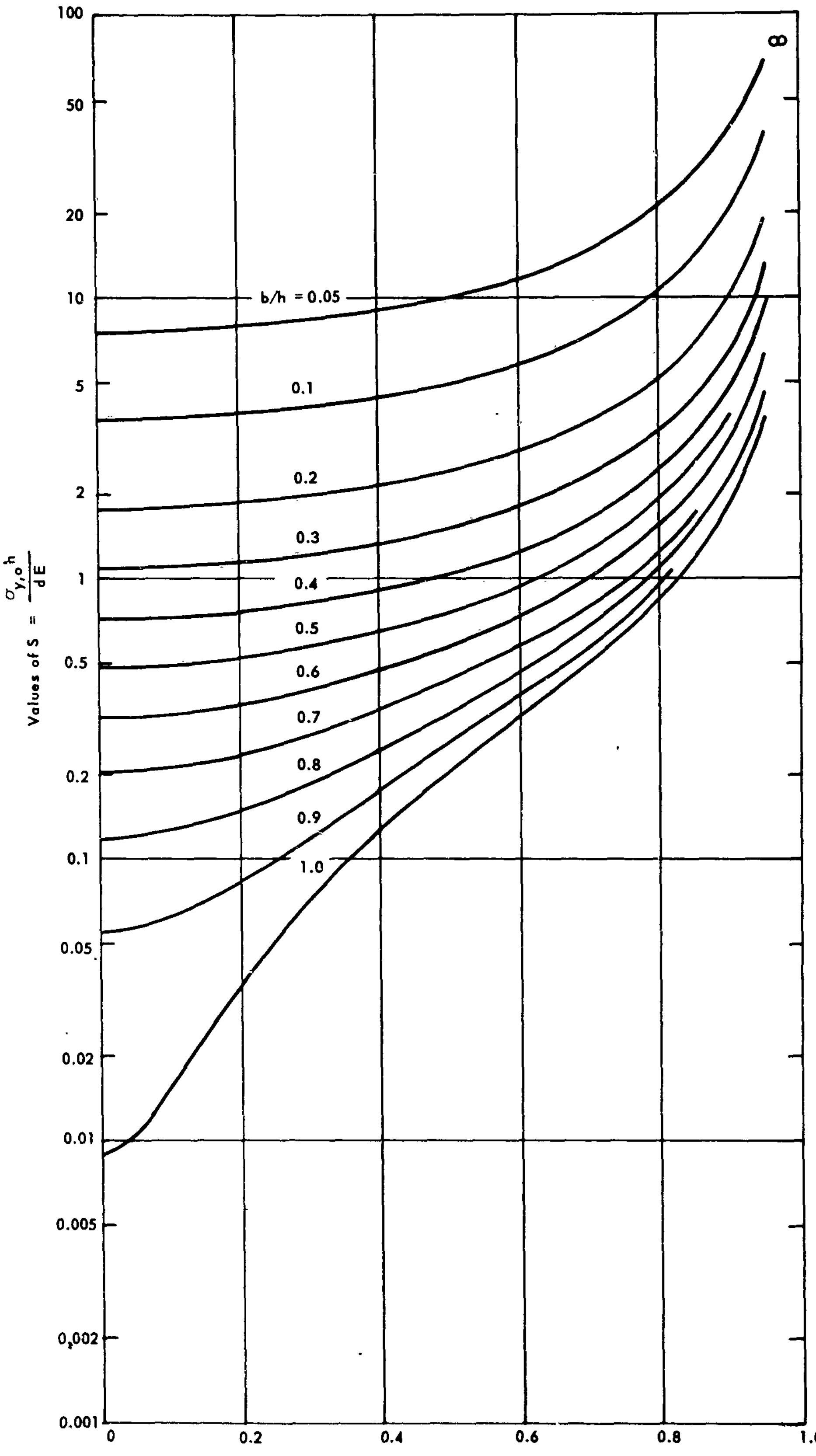

Figure 3. Pressure distribution on the base for μ = 0.25.

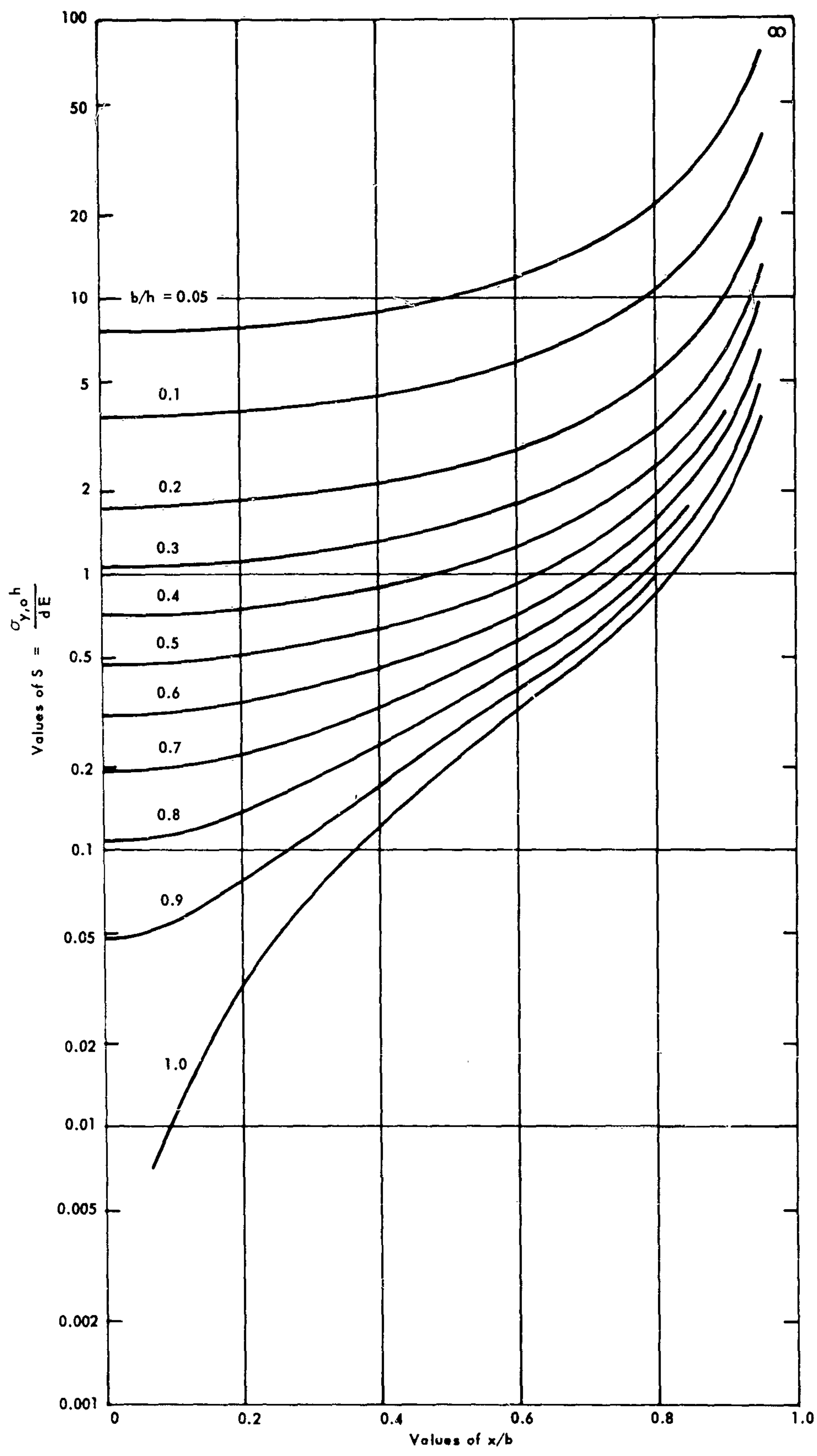

Figure 4. Pressure distribution on the base for μ = 0.3333.

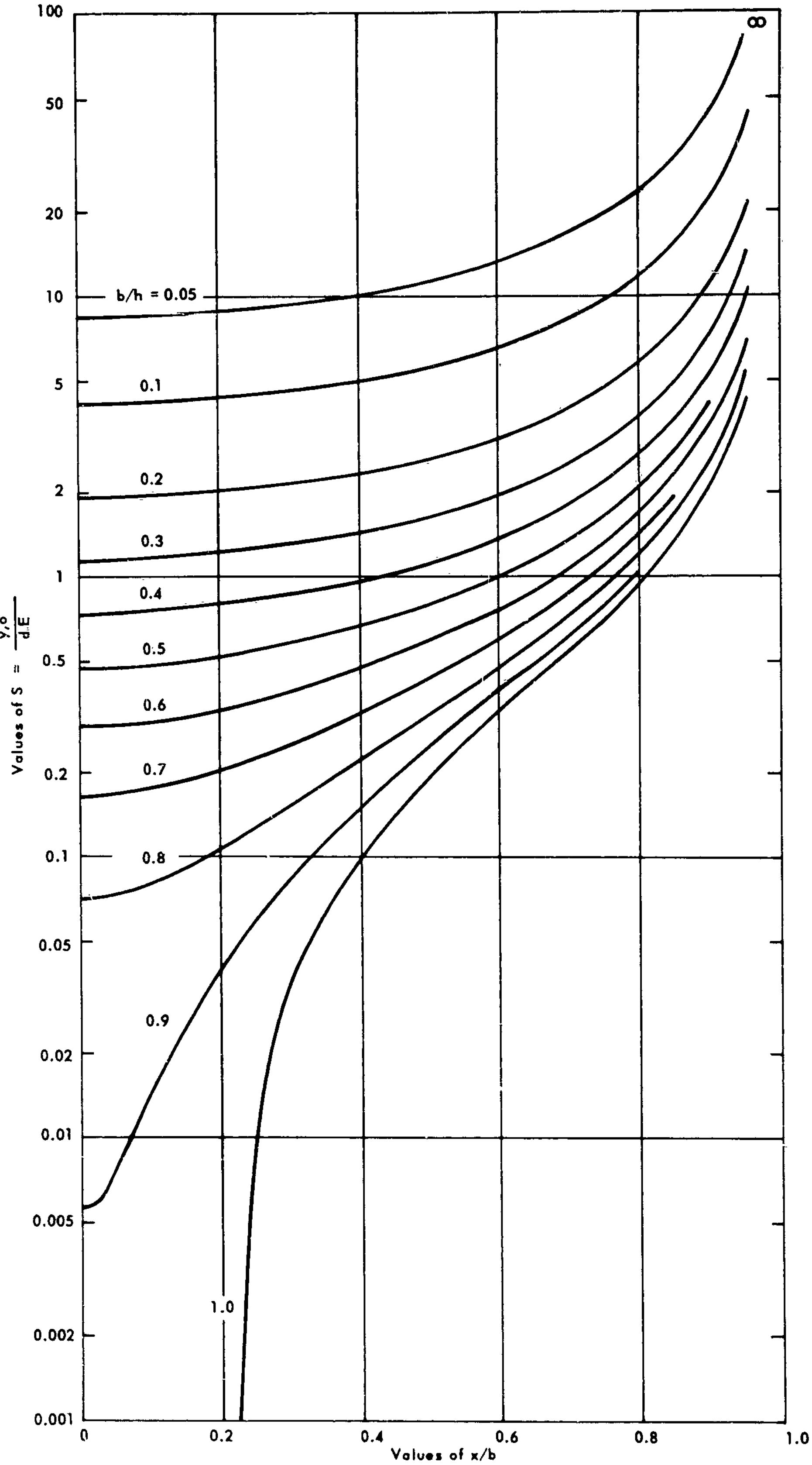

Figure 5. Pressure distribution on the base for μ = 0.5.

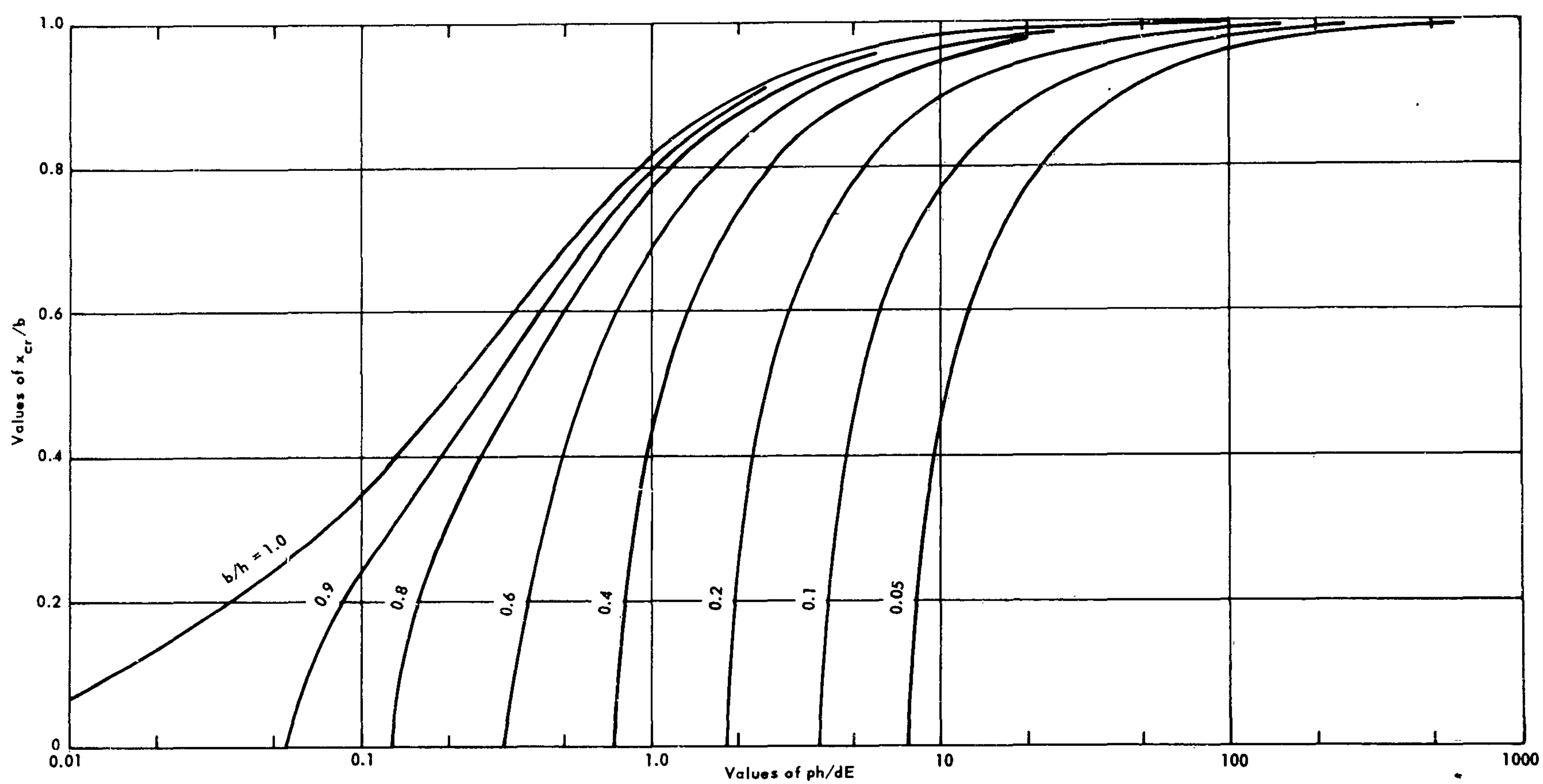

Figure 6. Distance to point of zero pressure on the base for μ = 0.1.

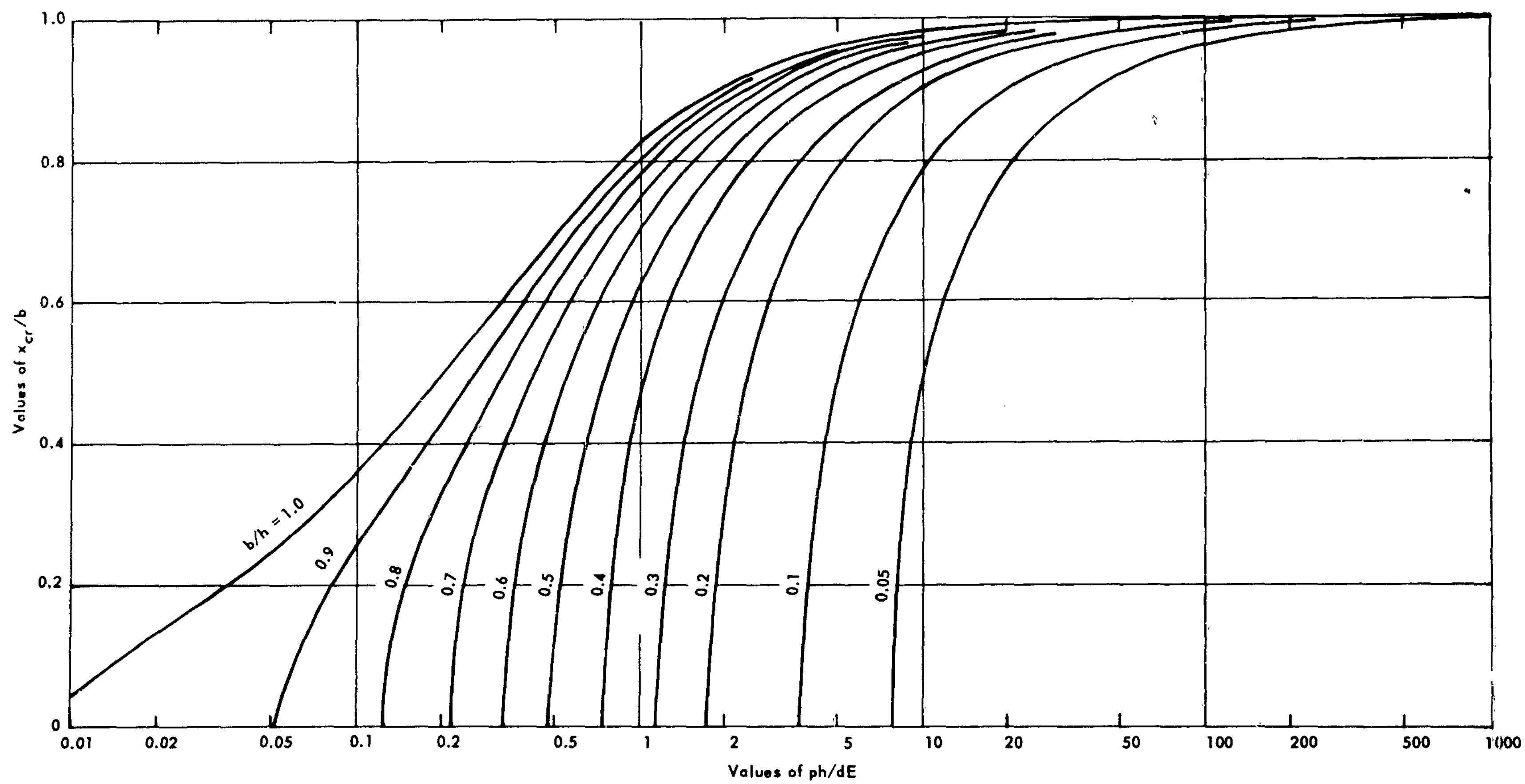

Figure 7. Distance to point of zero pressure on the base for μ = 0.25.

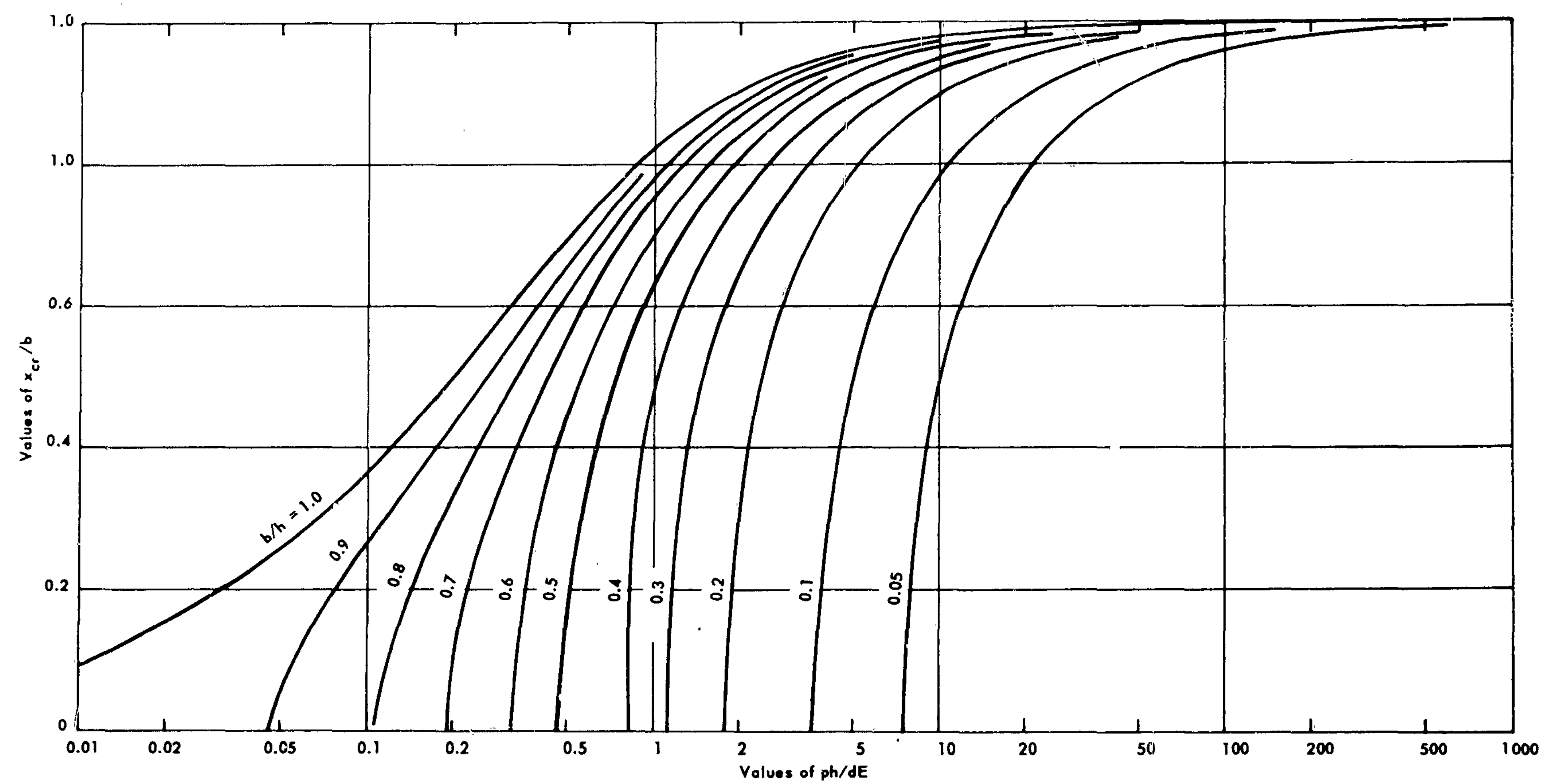

Figure 8. Distance to point of zero pressure on the base for μ = 0.3333.

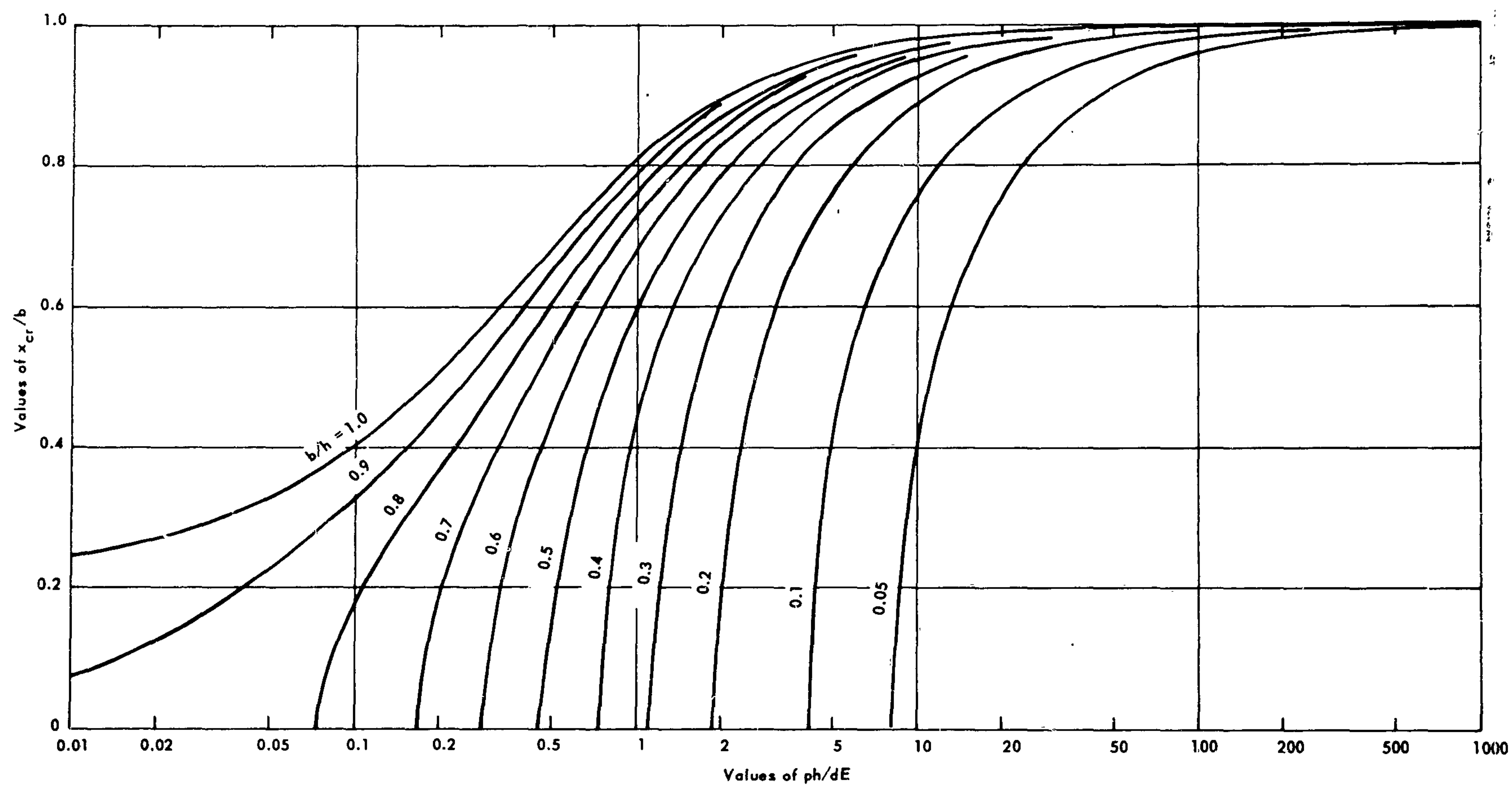

Figure 9. Distance to point of zero pressure on the base for $\mu = 0.5$.

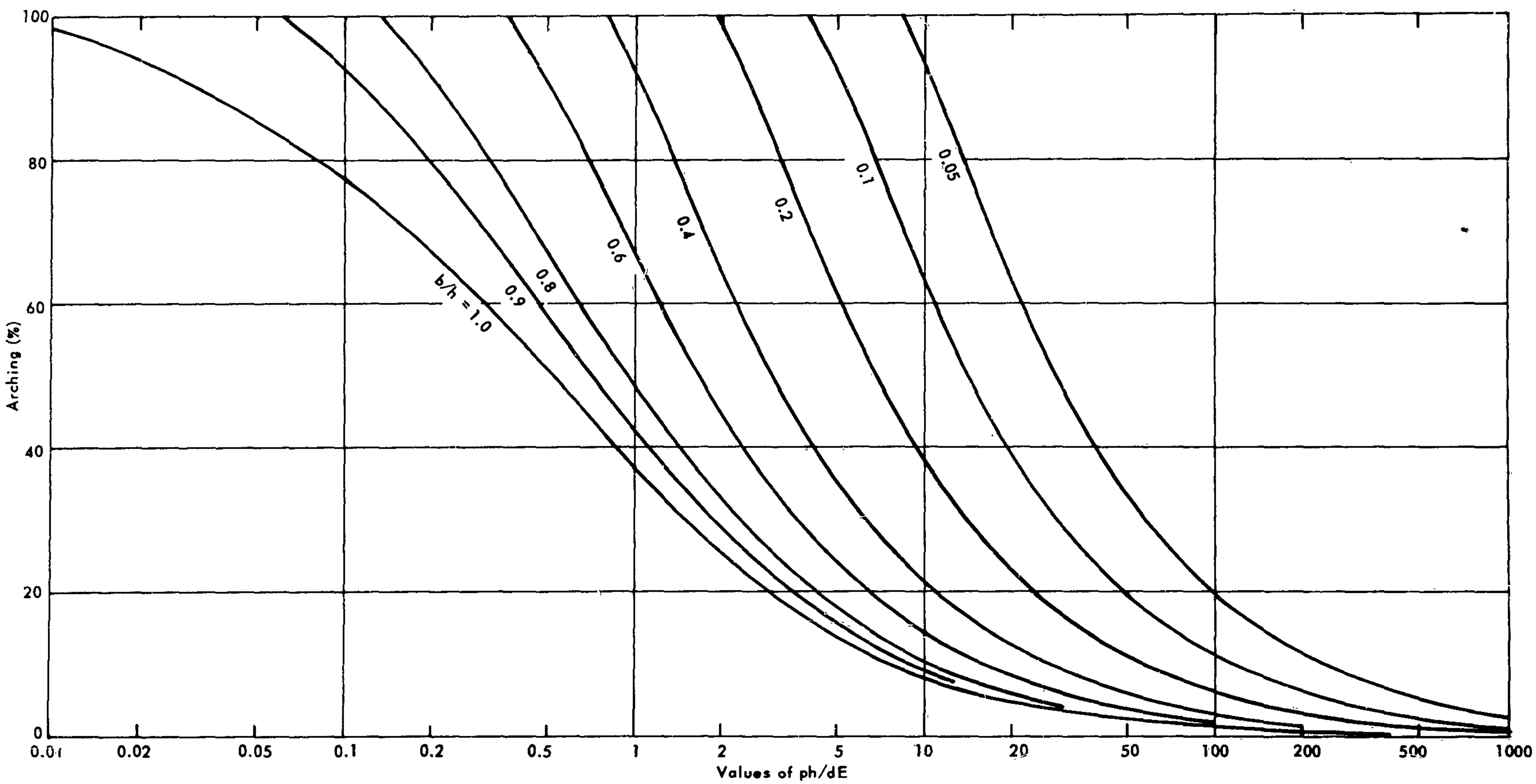

Figure 10. Arching due to deflection of the base for $\mu = 0.1$.

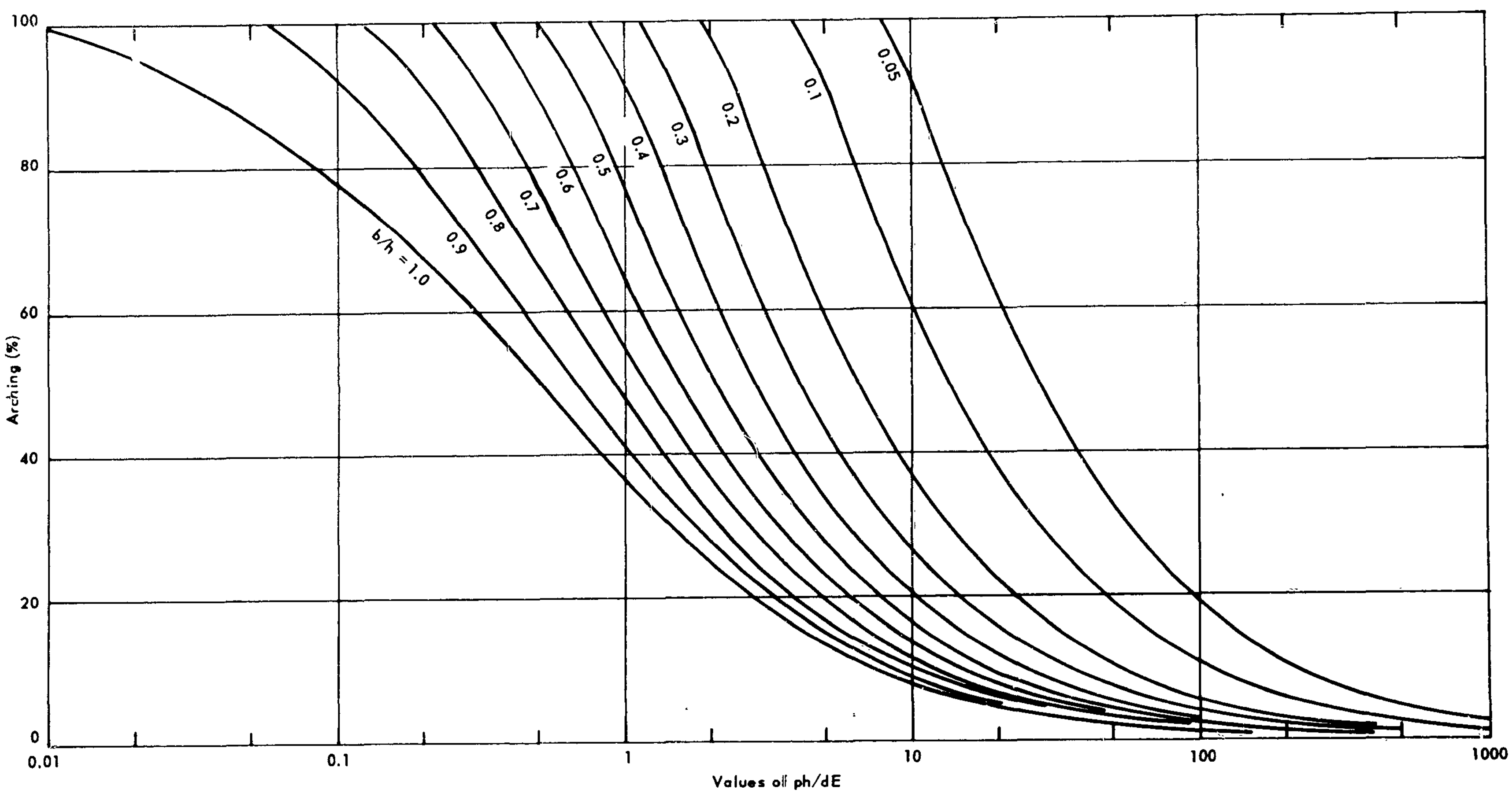

Figure 11. Arching due to deflection of the base for μ = 0.25.

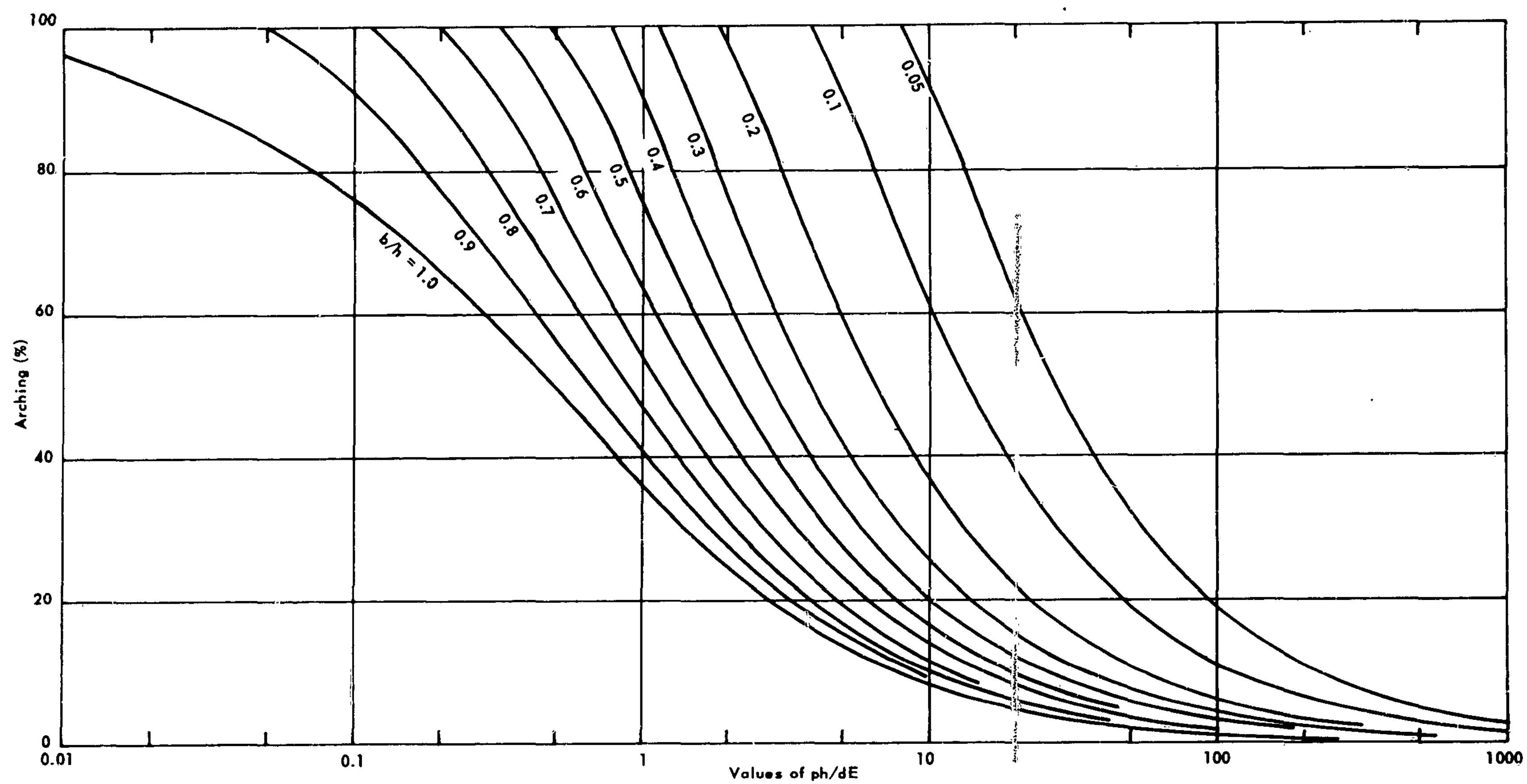

Figure 12. Arching due to deflection of the base for $\mu = 0.3333$.

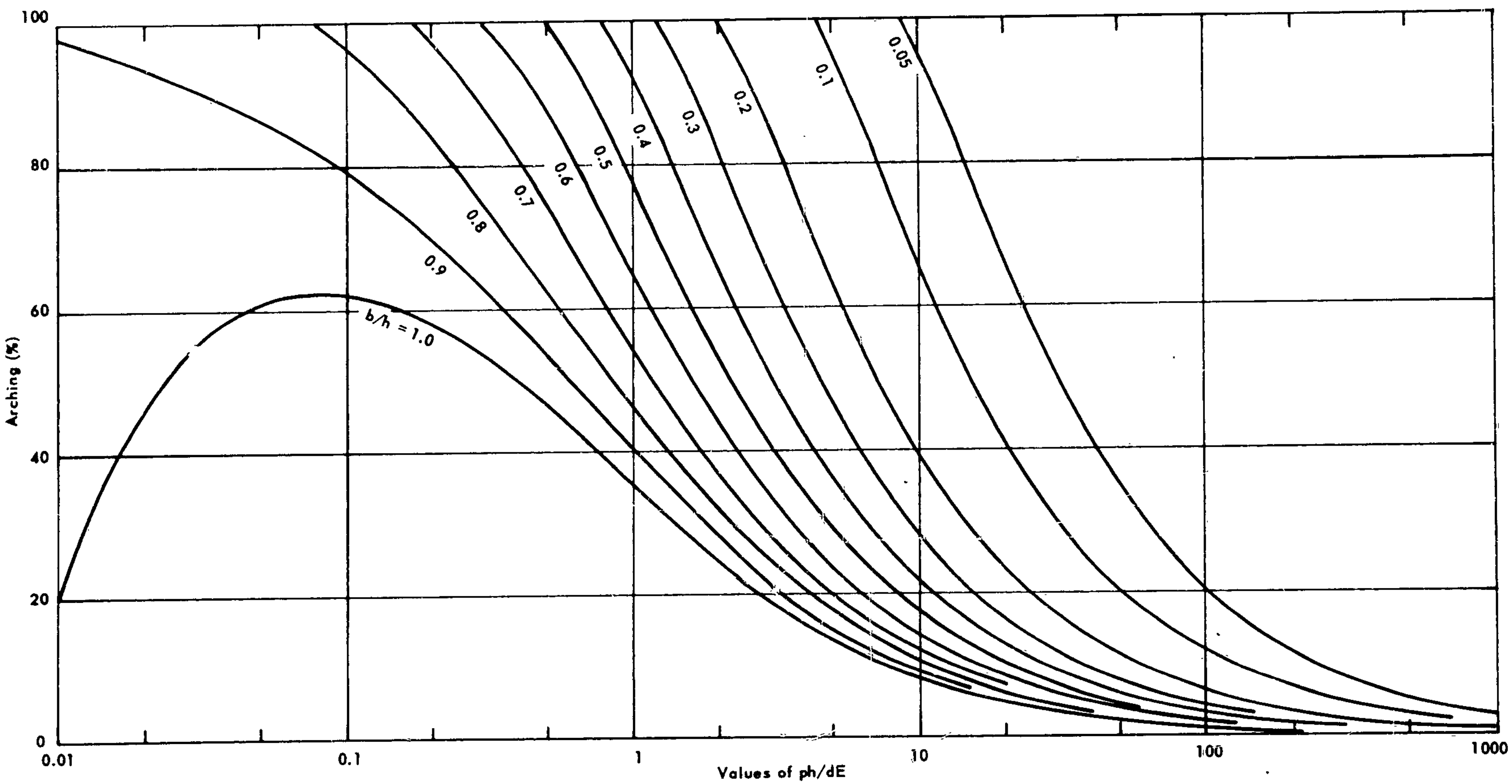

Figure 13. Arching due to deflection of the base for μ = 0.5.

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1. ORIGINATING ACTIVITY (Corporate author) U. S Naval Civil Engineering Laboratory Port Hueneme, California	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b GROUP

3. REPORT TITLE

Arching in Soil Due to the Deflection of a Rigid Horizontal Strip

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Final report

5. AUTHOR(S) (Last name, first name, initial)

Chelapati, D. C.

6 REPORT DATE	7a. TOTAL NO OF PAGES	7b. NO. OF REFS
October 1964	71	4

8a CONTRACT OR GRANT NO. P. O #110 b PROJECT NO. Y-F008-08-02-108a c DASA 13.018 d.	9a ORIGINATOR'S REPORT NUMBER(S) N-591 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

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A study is made of the amount of arching developed in an ideal soil of finite depth due to the deflection of a rigid horizontal strip or base when the soil is subjected to high static pressures. Solutions are obtained using the equations of plane strain in the form of an infinite series. A condition is imposed that the net pressure on the base cannot be tensile. It is shown that arching in this case is a function of the parameters b/h, ph/dE, and μ; where 2b is the width of the base, h is the depth of soil, p is the pressure on the base with no deflection, d is the amount of displacement, E is the modulus of elasticity, and μ is Poisson's ratio of the soil. The first six terms in the series are evaluated using a digital computer for a wide range of parameters. The accuracy of the final solution is shown to be quite adequate. Graphs are presented showing the pressure distribution on the base and the amount of arching over the base, and an example is given to demonstrate the use of these plots.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
arching	7, 8					
soil	9					
soil-structure interaction	8					
trap-door problem	8					
deflection	6, 7					
loads (forces)	6					

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